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Differences in the categorization of physics problems by novices and experts

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Differences in the categorization of physics problems
by novices and experts

by

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CHAPTER I. INTRODUCTION

Problem solving is one of the experiences common to individuals. Illustratively, starting an automobile on a cold January morning, maintaining good relationships with co-workers, finding the most advantageous mortgage terms, planning a career change, and determining the identity of an unknown ion in a chemistry experiment involve problem solving.

This study constitutes a specific inquiry into the categorization of physics text problems. In this context a physics problem is viewed in a manner similar to that of Newell and Simon (1972) in their description of a problem in general:

> To have a problem implies (at least) that certain information is given to the problem solver: Information about what is desired, under what conditions, by means of what tools and operations, starting with what initial information, and with access to what resources (p. 73).

This is not to say that the statement of any physics problem explicitly contains each element of the foregoing description of a problem. For clarification, consider the physics problem:

> A girl (mass M) stands on the edge of a merry-go-round (mass 10 M, radius R, rotational inertia I) that is not moving. She throws a rock (mass m) in a horizontal direction, tangent to the outer edge of the merry-go-round. The speed of the rock relative to ground is V. Neglecting friction. what is the angular speed of the merry-go-round? (Halliday & Resnick, 1974, p. 208a).

The question, constituting the last sentence in the problem statement, contains what is desired (angular speed of the merry-go-round) and the condition (neglecting friction). The tools and operations (applicable equations and subsequent algebraic manipulations) and access to resources (conservation of angular momentum and the correct expression for rotational inertia obtained from accessing domain-dependent knowledge in the cognitive structure of the problem solver and/or a physics text) are not included in the problem statement. The initial information is stated explicitly within the first three sentences of the problem statement.

The problem-solving process as described by Newell and Simon (1972) includes, as one of the initial steps, the formation of an internal representation of the external environment. This internal representation provides the framework within which the problem is to be solved. It follows directly that the representation formed by the problem solver determines whether and how the problem can be solved. The problem solver operates on the representation rather than the statement of the problem. Chi, Feltovich, and Glaser (1981) define a problem representation as a cognitive structure, corresponding to a problem, that is constructed by a solver on the basis of domain-related knowledge and its organization.

Achievement in mathematics and sciences such as chemistry and physics is heavily dependent upon the ability to solve problems. Physics assignments often consist of problems to be solved. Students competing in the Tests of Engineering Aptitude, Mathematics, and Science, a high school academic competition sponsored by The Junior Engineering Technological Society, may take a physics examination as one of the available options. Sixty-five percent of the forty questions in a recent physics test (JETS, 1985) are problems to which one correct, numerical solution exists. A few other items in this test require the student to solve non-numerical problems. Greeno (1978) claims that the strengthening of students' skills in problem solving is a major objective of mathematics and science instruction and believes that students must at least acquire the specialized knowledge they need to solve problems in the domain of the course.

Clearly, instruction in physics that improves the problem-solving processes of physics students is beneficial to them.

Teachers have observed that problem solving is often viewed to be a difficult process by their students and often find the teaching of problem solving to be very demanding. Physics students occasionally tell their teachers that they understand the text but are not able to solve the associated problems. Champagne and Klopfer (1981) indicate that a remarkable degree of agreement regarding the important role of problem solving in the learning of science exists among science educators. They refer to the National Assessments of Education Progress (1977) results for science which indicate that deficiencies in higher-order mental skills such as analysis, synthesis, and evaluation occur in a majority of students of ages thirteen and seventeen. Since these skills are components of problem-solving ability which, in turn, is important to the learning of science, learning outcomes recommended by science educators do not result as often as is realistically desirable. Larkin and Reif (1979) view the task of teaching students to become proficient problem solvers not only to be crucially important but also difficult. They believe that most students experience less difficulty in acquiring a knowledge about science than in learning the flexible application of this knowledge to the solution of diverse problems. These researchers substantiate the experiences of teachers and students that surface in casual conversations: Problem solving is difficult to learn and to teach.

Purpose and the Research Question

Previous learning affects later learning (Ausubel, 1968; Gagne, 1977). Much of the problem-solving research has been done in physics, and more particularly in mechanics since it, while being sufficiently complex, is based on a relatively small number of principles and has a mathematical structure. Prior knowledge affects the comprehension of physics principles (Champagne, Klopfer, & Anderson, 1980; Champagne, Klopfer, & Gunstone, 1982; diSessa. 1982). Heller and Reif (1984), Larkin (1980), and Chi, Feltovich, and Glaser (1981), have found that the representation formed by the problem solver, based on domain-dependent knowledge, is a crucial step in the problem-solving process.

Champagne, Klopfer, and Gunstone (1982) conclude that:

> Preliminary qualitative analysis of physics problems is seldom if ever taught explicitly. In fact, problem-solving instruction in physics textbooks makes no attempt to link the physical features of the real-world situations described in physics to abstract concepts and principles of the Newtonian framework (p. 37).

The linkage of such features becomes part of the representation of problems. Chi et al. (1981) believe that the categories that problem solvers impose on physics problems represent organized knowledge structures in memory (schemata) that determine the quality of the representation process. Research that increases the understanding of categorization, and thus representation, may eventually influence the design of instructional materials and strategies.

This research is designed to answer the question: "Do novices and experts differ in the categorization of physics problems?" Operationally the study investigates differences in the categorization, important to the representation of physics problems, that are believed to exist between novices and experts. Chi, Feltovich, and Glaser (1981) claim that the categorization imposed on physics text problems by problem solvers and concomitant representations formed by them reveal differences between novice and expert physics problem solvers. This study, as does the Chi et al. (1981) research, requests subjects to categorize sets of mechanics problems using a sorting procedure. The categories are based on similarities of solutions that would occur if the subjects were to solve the problems. The subjects do not actually solve the problems in order to form the categories but express the reasons for their selection of the categories in written form. Chi et al. (1981) found that subjects with greater amounts of physics knowledge categorize primarily according to deep structures, i.e., physics laws and concepts. Subjects with lesser knowledge key on surface structures, i.e., objects such as springs, pulleys, and levers, specific physics terms such as friction, and spatial arrangements.

Objectives

1. To verify that subjects with different degrees of physics knowledge differ in the categorization of physics (mechanics) text problems: Experts categorize according to deep structures and novices classify according to surface features. The subjects will sort a problem set consisting of most of the problems used by Chi et al. (1981).
2. To initiate the generalization of expert-novice differences in categorization beyond a specific set of problems: To determine whether expert-novice differences are independent of the set of problems used in Objective 1. The subjects will sort a set of physics (mechanics) text problems different from that used in Objective 1.
3. To test the research outcome of experts categorizing according to deep structures regardless of surface features, novices categorizing according to surface features regardless of deep structures, and intermediates revealing a categorizing pattern that is characterized by a mixture of deep structures and surface features (Chi et al., 1981). The subjects will sort a problem set in which surface

features are specifically counterbalanced by deep structures. The problem set is different from the problem set used by Chi et al. (1981) in their work on this objective but contains some problems common to the two sets.

4E. To determine whether experts (E) categorizing a set of physics (mechanics) problems that contains four deep structures and four surface features construct twice as many categories as occur when they categorize a set that contains two deep structures and two surface features. The subjects will sort a set of specifically counterbalanced problems that contains two deep structures and two surface features. The problem set used in Objective 3 serves as the comparison set. It contains four deep structures and four surface features.

4N. To determine whether novices (N) categorizing a set of physics (mechanics) problems that contains four different surface features and four deep structures construct twice as many categories as occur when they categorize a set that contains two surface features and two deep structures. The subjects will sort a set of specifically counterbalanced problems, also used in Objective 4E, that contains two surface features and two deep structures. The problem set used in Objective 3 serves as the comparison set. It contains four deep structures and four surface features.

Hypotheses

The hypotheses are derived directly from the preceding objectives.

1. Experts will categorize physics (mechanics) problems on the basis of deep structures and novices will categorize these problems on the basis of surface features.
2. Experts will categorize a different set of physics (mechanics) problems on the basis of deep structures and novices will categorize this set on the basis of surface features.
3. Experts will categorize physics (mechanics) problems according to deep structures regardless of surface features and novices will categorize these problems according to surface features regardless of deep structures. Intermediates will reveal a categorizing pattern that is characterized by a mixture of deep structures and surface features.

4E. Experts (E) will categorize a set of physics (mechanics) problems according to deep structures regardless of surface features with the number of established categories approximately equal to the number of deep structures contained within the set.

4N. Novices (N) will categorize a set of physics (mechanics) problems according to surface features regardless of deep structures with the number of established categories approximately equal to the number of surface features contained within the set.

Definition of Terms

Deep structures - Included in the problems used in the sorting tasks: physics laws and concepts used in the categorization and/or representation of physics problems.

Expert - Physicist, holding the Ph.D. degree in physics, who has taught physics courses in a college or university.

Intermediate - Person who has completed more than a one-year physics course but less than a B.S. degree in physics; person intermediate between novice and expert .

Novice - Person who has completed the mechanics portion of a first-year physics course but who has not begun a physics course beyond this first-year course.

Problem - Situation in which the problem solver has certain information about what is desired, under what conditions, by means of what tools and operations, starting with what information, and with access to what resources (Newell & Simon, 1972).

Representation - Schema by means of which the problem solver describes the environment and solves a problem by mental operations on this description; a transformation of presented information.

Schema - Organized knowledge structure within memory that contains knowledge about a concept. According to Gagne (1985): Includes static qualities (structures), active qualities (expectancy toward information), conscious use (for example, retrieval guidance), and automatic use (for example, recognition of a new instance of a concept). Operationally, as used by Andre (1986): Representation of concepts (categories), principles/rules (relationships between concepts), and skills (activities requiring several steps). pl.: Schemas, schemata.

Surface features - Included in the problems used in the sorting tasks: Objects such as levers, springs, and pulleys; specific physics terms such as friction and force; spatial arrangements.

Assumptions

1. One qualitative and/or quantitative answer to each physics problem in the study is assumed to exist although solutions may differ.
2. Newell and Simon (1972) do not rule out that learning occurs during tasks that last tens of minutes. It is assumed that the extent of learning that may occur during the sorting tasks does not affect the performance states of the subjects.
3. Research-based knowledge about similarities such as Aristotelian concepts in the cognitive structure of students can serve as a basis for the design of instructional materials such as the interactive software by Champagne and Klopfer (1982). The links between problem categorization/representation and the design of actual instructional material are

not known: This is a particular case of the well-known difficulty of applying research findings to classroom practice. However, if categorization and representation are important to the problem-solving process, it follows that knowledge concerning such categorization and representation will eventually be embodied in instructional materials and strategies.

Delimitations

Problem solving is often associated with learning. Novak (1976) believes the ability to solve relevant but novel problems to be the test for determining whether meaningful learning has occurred as he views problem solving to be a kind of meaningful learning within the perspectives of Ausubel's learning theory.

This research involves only mechanics problems and it is not assumed that the results are equally applicable to areas such as thermodynamics or quantum mechanics.

This study is concerned with the performance of individuals rather than their learning processes and relegates to problem solving the kind of place as is done by Newell and Simon (1972):

> This study is concerned with thinking - or that subportion of it called problem solving - but it approaches the subject in a definite way. It asserts specifically that thinking can be explained by means of an information processing theory (pp. 4, 5).

Studies in problem solving often involve small numbers of subjects.

The numbers of subjects in studies by Simon and Simon (1978), Larkin (1980), Chi et al. (1981), and Heller and Reif (1984) accomplishing a given task, are, respectively, one, six, eight, and twenty-four.

The small numbers of subjects in various problem-solving studies occur, in part, because of the application of information processing (considered in the beginning of Chapter II) to psychology. As expressed by Newell and Simon (1972):

> The technical apparatus for conceptualizing information processing systems leads first of all to constructing particular programs that accomplish particular tasks. When applied to psychology, this procedure leads naturally to constructing information processing systems that model the behavior of a single individual in a single task situation. Full particularization is the rule, not the exception. Thus it becomes a problem to get back from this particularity to theories that describe a class of humans, or to processes and mechanisms that are found in all humans (p. 10).

In the information-processing model, the human being is viewed as a complex mechanism with constituents that can eventually be understood in detail. The individual human is considered as a "world" that in itself provides the realm of the investigation and this "world" may be studied in a small number of subjects.

A second reason for the small number of subjects involved in these studies is related to the full particularity just mentioned. The detailed and multi-faceted nature of the data within a complicated context that often results from these studies taxes the available time and resources of the researchers.

A reason interwoven with the preceding thoughts is that of the frequently limited availability of subjects such as Ph.D. physicists who are willing or able to devote the time necessary to complete tasks within this field of research.

The foregoing discourse does not preclude the use of other methods such as the analysis of variance, for which greater numbers are needed. Shulman (1980) suggests that the methods of analysis should be chosen carefully as he claims that the best research programs will reflect intelligent deployment of a diversity of research methods applied to their appropriate research questions: Methodologies may be combined in worthwhile studies.

This study utilizes cluster analysis for the treatment of categorization data. Cluster analysis is used to arrange a set of items (problems in this study) into subsets (clusters) so that items within a cluster have a high degree of homogeneity when compared to items from different clusters. A dendogram, a graphical representation that may be derived from association matrices, is used to present cluster analysis information. This research, with only partial reliance on the well-known quantitative techniques, strives for a measure of generalizability within the context of cognitive work in problem solving.

CHAPTER II. REVIEW OF LITERATURE

What occurs in the mind? How does learning occur? What are the components of problem solving? These questions touch upon human intelligence and may be treated in cognitive psychology. Intelligence and its site, the brain, have aroused man's curiosity for hundreds of years. The brain is of such complexity that the understanding of thought presently cannot rest on physiological or chemical studies: Billions of neurons participate in a vast number of chemical reactions involving complex substances. Even if the functions of each neuron were known in detail, the resulting explanation would be immensely cumbersome. Presently cognitive psychologists use the information-processing approach rather than a physiologically-based approach in problem-solving research.

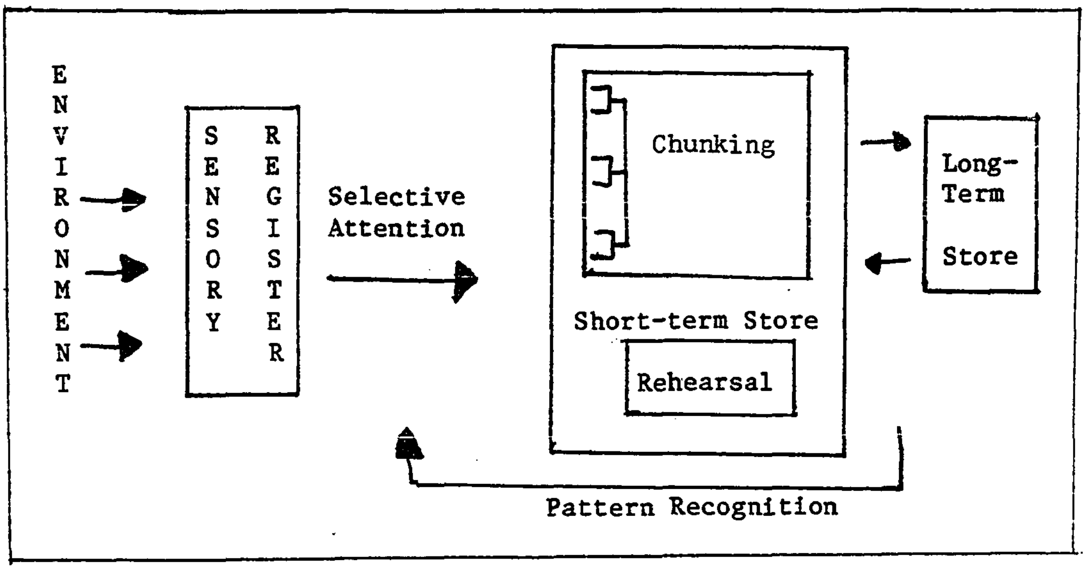
Similarities between human thought processes and those occurring in computers have contributed to practical acceptance of this approach. High-level languages such as BASIC and LOGO make use of statements that are converted to machine-level instructions as a computer carries out a specified task. The higher-level language may be used to describe the behavior of the computer without resorting to machine language. Cognitive psychologists use constructs for the description of intelligence without dependence on a detailed mapping of structures and functions of the brain.

Nature of the Cognitive System

The information-processing model has a degree of abstraction that precludes the necessity of associating specific architectural components with particular physical parts of the brain. Thus, in a sense, one can think of the architecture of the human information-processing system independent of the consideration of the physical brain.

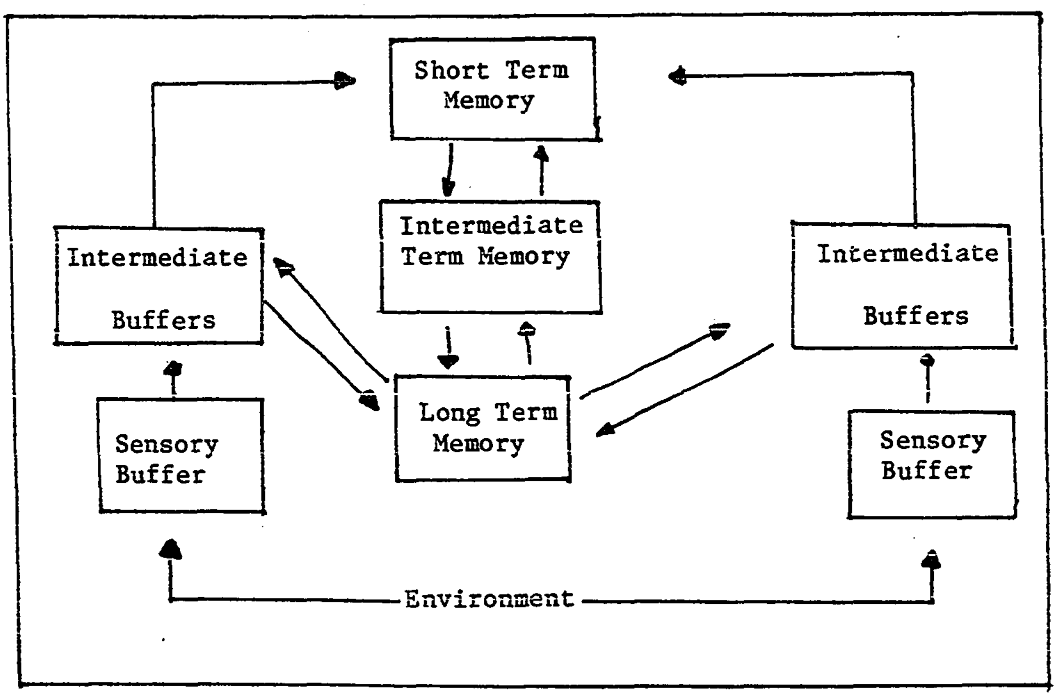
Atkinson and Shiffrin (1968) proposed a model including the sensory register, short-term store (memory), and long-term store (memory). Figure 1 facilitates a brief discussion of the human information-processing system. Not all cognitive psychologists accept the same model; Figure 2 depicts the Distributed Memory Model (Hunt, 1971, 1973) that also assumes the existence of different memory areas in the brain. Information is distributed into these memory areas and may be transferred among them as indicated in Figure 2. The models, as does the mind, receive, change, store, retrieve, and use information. The models are different, but each contains the sensory register (buffer), the short-term store (memory), and the long-term store (memory), and allow for the tracing of information through the system.

The flow of information is mostly governed by processes that are controlled by the individual who, as such, assumes an active role in thinking, or more specifically, problem solving. In Figure 1, environmental stimuli (informational inputs) enter the sensory register and some of them are changed to an internal code. The sensory register retains the coded information for a very brief interval (fractions of seconds). The expectancy of an individual partially determines which of



(DuBois, Alverson, & Staley, 1979)

Figure 1. Information-processing model of human memory



(Hunt & Poltrock, 1974)

Figure 2. The distributed model of human memory

the various stimuli eventually are stored, in changed form, for later retrieval and use. The individual's motivation, prior learning and experiences, and instructions received by the individual give rise to this expectation which is a kind of awareness or anticipation that certain events are likely to occur in a given situation. Different individuals receiving the same stimuli, because of different schemata in long-term memory (LTM) which affect pattern recognition that is brought to bear on information in the sensory register, differ in the information that they allow to be passed into short-term memory (STM). In Figure 2, the environmental stimulus moves through the intermediate buffers. Networks in LTM affect information in the intermediate buffers. Interactions between LTM and the intermediate buffers continuously and hierarchically develop the information until it, in the form of complex codes, enters STM where it remains for a few seconds unless it is rehearsed.

Both models provide for the transmission of information from STM to LTM. The first model proposes chunking (reorganization of information) and rehearsal in order to increase selectively the amount of information that is retained in STM. Rehearsal and elaborative encoding facilitate the transfer of information from STM to LTM. Not all rehearsal results in such transfer. Maintenance rehearsal tends to keep information in STM, also called the working memory, without increasing the informational content of LTM. Not all models include a store that may alternatively be called STM or working memory: The model proposed by Greeno (1973) includes a working memory separate from but interacting with STM.

The information that is to be learned can be modified by elaborative encoding. The altered representations that result are then stored in LTM. According to Paivio (1971) there are two kinds of memory representation (elaborative encoding strategies). The imagery system stores information with analogical keys to perceptual qualities such as smell and tone while the verbal system allows information to be stored in an abstract linguistic form that relates to objects and actions in an arbitrary fashion. Retention of information is increased when it is encoded in both systems, i.e., dual encoding. Elaborative encoding changes information that is to be learned to a form that is more meaningful to the individual and may be of the semantic type in which a verbal mediator may be formed. For example, the expression "cks" might be encoded into a meaningful word such as "crankshaft" and stored in that form. Later the word may be retrieved and recoded to "cks." Organization of the content to be stored is another form of semantic encoding.

People tend to cluster information in an effort to increase memory. The capacity of STM is considered to be what Miller (1956) called "the magical number seven plus or minus two," i.e., the number of items that can be recalled in order immediately after an introduction to a list. The amount of information held in STM is important as it is available for encoding and transfer to LTM.

The Distributed Memory Model includes an intermediate-term memory (ITM) that contains developing ideas for periods on the order of several minutes to hours before being transferred to LTM. Hunt and Poltrock (1974) claim the existence of ITM on logical grounds rather than empirical evidence.

Both models include LTM as the area of permanent and virtually limitless storage. The permanence of this stored information implies that forgetting is tantamount to being unable to access such information. The framework in LTM may be compared to a library in which books are stored and classified or to the permanent memory of a computing system. Both analogies have in common that information is stored and retrieved in certain ways.

Models of the human information-processing system, while not identical, provide an abstract "place" within which activities such as the solving of physics problems occurs.

Much of cognitive learning theory is related to the information-processing approach.

Cognitive Learning Theory

The recurring theme of later learning being affected by previous learning rests on the belief that conditions for learning must exist within the cognitive structure of the learner. Gagne (1977) groups these internal conditions into general prerequisites that allow for new learning to occur and specific prerequisites that become incorporated into new learning. Examples of general prerequisites are reading and memory strategies. Knowing how to differentiate in order to obtain an acceleration expression from a velocity expression is an example of a specific prerequisite. Gagne recognizes five learning domains: Motor skills, attitude, verbal information, intellectual skills (mental operations such as discriminations and rules performed on internal representations), and cognitive strategies.

Dubois, Alverson, and Staley (1979) reclassify Gagne's domains in order to bring about a better conformance with the taxonomy of learning as constructed by Bloom (1971). The reclassified domains are the psychomotor domain, the affective domain, the cognitive domain (Gagne's verbal information and the learning of concepts and rules from the intellectual skills domain), and the mathemagenic domain. Rothkopf (1970) introduced the word "mathemagenic" in the phrase "mathemagenic activities" to describe those activities that largely determine what people learn as they interact with instructional materials. Dubois et al. (1979) use "mathemagenic activities" to describe all strategies that the learner uses in the learning of new material. In this context, tasks that involve mathemagenic skills are viewed as problem-solving situations in which prior knowledge and mathemagenic skills are applied. Mayer (1980) describes the mathemagenic domain in terms of behaviors produced by the learner during the course of learning that influence the learning of the material under consideration.

In a sense, to live is to deal with a continuous sequence of problems. Problem solving includes a myriad of components such as necessity, motivation, curiosity, tenacity, and rewards. The research in problem solving has not dealt with all kinds of problems in life but has centered on cognitive rather than affective elements. The problem-solving process occurs primarily within the cognitive domain. The mathemagenic domain is also involved because of the importance of problem solving to the learning of subjects such as chemistry and physics. Cognitive learning theories such as Gagne's allow for the ascription of certain mental activities to domains. Such theories attempt to explain how learning occurs. Prior learning is brought to bear on the problem-solving process. Because of such connections between learning and problem solving, cognitive knowledge concerning problem solving is relatable to a learning-theory framework. Thus, while the purpose of this study involves problem categorization, cognitive learning theory and developments in problem-solving research form a consonant relationship.

Ausubel's cognitive learning theory (1968) predated much of the current cognitively oriented research in problem solving. This theory is selected here to illustrate the compatibility of notions such as schemata as used in learning theory with their use in problem-solving research: Schemata are organized structures in memory that contain knowledge about concepts. The choice of Ausubel's work was made because this theory provides a framework of relating new information to existing information in cognitive structure as concepts are learned. Concepts, in this perspective,

> ...describe some regularity in relationship within a group of facts and are designated by some sign or symbol. Thus red is a concept describing the regularity of color, but the label "red" is also used to describe a regularity in the political stance of an individual or group (Novak, 1977, p. 18).

This meaning of the word "concept" is compatible with the meaning used in much cognitive work.

Piaget's cognitive developmental theory of learning is often discussed and presented through the writing of others who have studied his work in detail. Ausubel's theory is treated by Novak (1977) as he interprets Ausubel's theory in depth and Dubois, Alverson, and Staley (1979) as they present a brief summary of this work. This study draws upon these writings.

Meaningful learning is the fundamental idea in Ausubel's theory. Meaningful learning occurs when new information is related to knowledge possessed by the learner, i.e., incoming information is linked with the relevant schemata within the cognitive structure in LTM of the learner. The new information must have a certain form in order to be relatable:

> In order for meaningful learning to occur, the information itself must have certain qualities. Such information has logical meaning...the critical qualities it possesses are substantiveness and nonarbitrariness.
>
> Substantiveness means that the information is capable of being paraphrased without changing the idea expressed. If the idea expressed in the information has a nonarbitrary relationship to another idea, then the information is said to possess "nonarbitrariness" (Dubois, Alverson, & Staley, 1979, pp. 132, 133).

Consideration of an example may be helpful. The statement "teachers are learners" may be stated as "teachers are people who show new behaviors in a given situation caused by their repeated experiences in that situation." Not only does the expression yield to paraphrasing without a loss in meaning but also relates teachers to learners through the use of "people" which is clearly a nonarbitrary relationship.

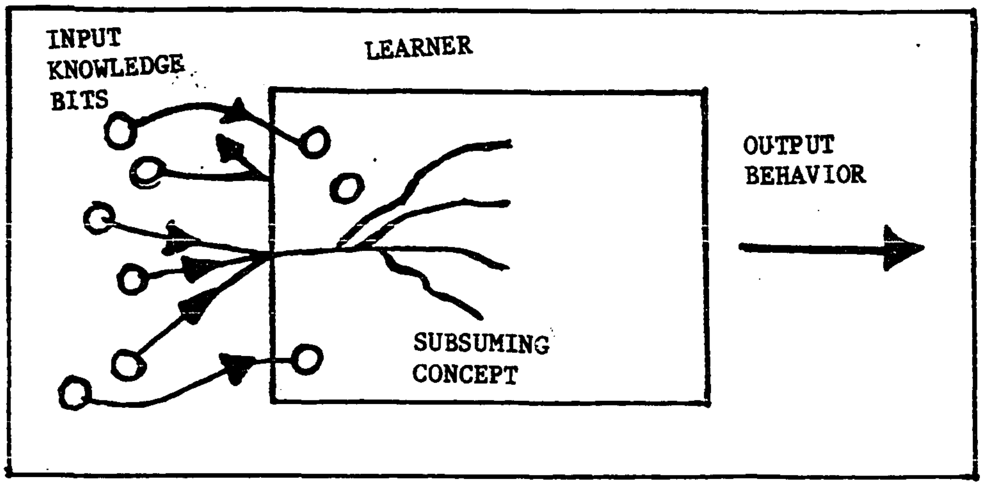
Concepts that are being acquired must be linked with previously-learned concepts. Ausubel uses the notion of a subsuming concept or subsumer as a mechanism for envisioning and explaining such linkage.

> ...the role of a subsuming concept in meaningful learning is an interactive one, facilitating movement of relevant information and previously acquired knowledge. Futhermore, in the course of this linkage, the subsuming concept becomes slightly modified, and the stored information is also altered somewhat.
> ...in the course of meaningful learning a subsumer becomes modified and differentiated further. Differentiation of subsumers results from assimilation of new knowledge in the course of meaningful learning (Novak, 1977, pp 82, 83).

Subsumption, formation of a relationship, may be superordinate, subordinate, or combinatorial (Ausubel & Robinson, 1969). Superordinate subsumption occurs when the existing schemata in LTM are less inclusive than the new information. A physics student may know that velocity involves direction, that pushing on the accelerator pedal of an automobile and releasing such a pedal have opposite effects, and that an applied net force on an object acts in the same direction as the subsequent acceleration. When this student now learns that such physical quantities include direction as well as magnitude (size) and are examples of vector quantities, superordinate subsumption has occurred. The resulting schema representing "vector" has been formed. The student may now observe an object in a physics experiment, determine its linear momentum (equaling the product of mass and velocity), and conclude that linear momentum is a vector quantity. Subordinate subsumption has occurred as the student recognizes that linear momentum is a particular instance of the concept "vector." The student may draw a vector diagram in order to add the momenta of two objects that collide during an experiment. A similar diagram may be drawn for the addition of the forces that act on the objects at a given time. Physical situations explained by vector diagrams and understood in terms of such diagrams illustrate combinatorial subsumption.

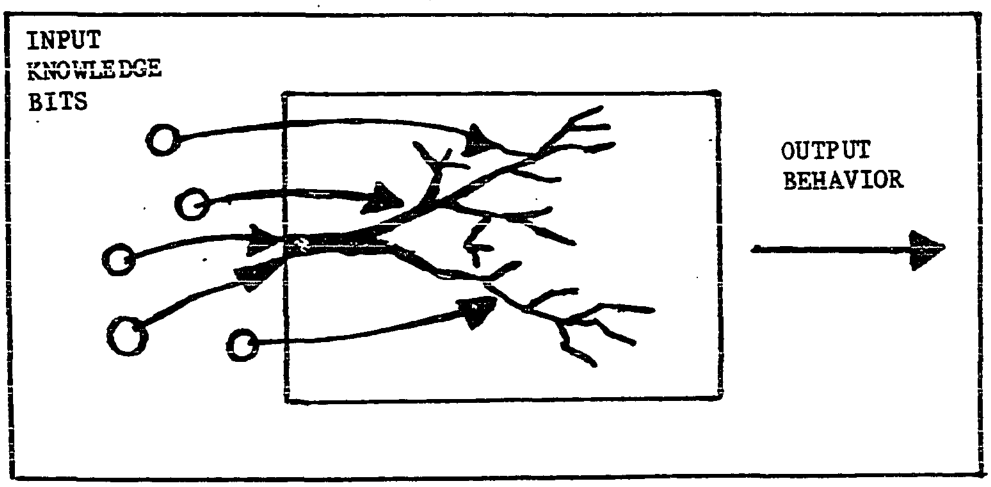
Information may be learned in a rote or meaningful fashion. Figure 3 (Novak, Ring, & Tamir, 1970) depicts rote learning as the acquisition of information that is stored independently as no subsumer exists. Information that can be related to a subsuming concept is learned meaningfully. Figure 4 (Novak et al., 1970) shows enlargement of the subsumer as further meaningful learning occurs. Novak (1977), noting that absolute rote learning probably only occurs in a newborn infant, stresses that rote learning and meaningful learning are related by means of a continuum. This, in turn, implies that meaningful learning increases with the size and number of the relevant schemata.

The work of Kuhn (1962) is employed by Champagne and Klopfer (1981) and Novak (1977) in dealing with learning research in science. Kuhn believes that the paradigms held by scientists largely determine which problems are researchable and also influence the choice of methodology used to investigate those problems. The shift from one paradigm to another is not accomplished easily but, when it does occur, increasing numbers of scientists accept the new paradigm. They then "see" the problem at hand differently and propose different solutions. Novak argues that Kuhn's social and conceptual frameworks not only guide



(Novak, Ring, & Tamir, 1970)

Figure 3. Information and subsuming concept



(Novak, Ring, & Tamir, 1970)

Figure 4. Changing a subsuming concept

scientific inquiry but all human inquiry and asks: "If science is recognized as evolving populations of concepts which guide both our methods of inquiry and our interpretations of our findings, should not science instruction also focus on concept learning?" (Novak, 1976, p. 496). Novak answers that question in the affirmative and advances Ausubel's theory as having the breadth necessary to address the epistemological and conceptual demands of learning research in science education.

Since learning partly depends on existing knowledge, cognitive knowledge theory is now explored briefly.

Cognitive Knowledge Theory

Newell and Simon (1972) call learning a second-order effect as it changes a system capable of certain performances into a system that can accomplish additional things. This changed system retains much of the pre-existing performance capability. This means that performance must be understood before learning processes can be studied in a theoretical context. It also means that there is a difference between performing a task and learning to perform a task. The purpose of this study, to investigate differences in the categorization of physics problems that are believed to exist between novices and experts, is related to performance. Thus it becomes important to focus on cognitive knowledge theory. To ask about the components of problem solving involves the ways in which information is stored in LTM. Yet it is reasonable to consider cognitive knowledge while concomitantly holding such knowledge against a learning theory backdrop as novices become experts through learning processes. A learning theory, at the very least, should be able to encompass the accumulating findings in cognitive research. A learning theory may guide learning research as Novak (1976) suggests and, because of the interdependence of learning and performance, the accumulating findings in problem-solving research may eventually be used in testing the theory and necessitate changes in its substance or interpretation. More specifically, because of the choice of Ausubel's theory in this study, definitions and concepts used in cognitive knowledge theory are expected to be appropriate for the description of the processes that relate new knowledge to existing schemata.

How does a novice problem solver differ from an expert problem solver in a given domain such as mechanics? Most answers to the question would probably include mention of a difference in knowledge. As mentioned earlier, Larkin and Reif (1979) believe that most students find that solving diverse problems is more difficult than acquiring knowledge about the science content that needs to be applied to the solving of such problems. Different kinds of knowledge seem to be involved as physics students give evidence of knowing about the principles essential to the solution of certain problems but are not able to solve them. Knowing "that" (being able to verbalize something) and knowing "how" involve different aspects of knowledge. A distinction between declarative knowledge (knowing "that") and procedural knowledge (knowing "how") may be used in a cognitive representation of knowledge.

Declarative knowledge may be represented by means of propositions.

Gagne (1985) defines a proposition as a basic unit of information that approximates a single idea. A proposition contains a set of arguments (one or more) and a relation: The arguments are the topics of the proposition and the relation constrains the topics. Arguments most often are nouns and pronouns but may be verbs and adjectives. Relations usually occur as verbs, adverbs, adjectives, and prepositions. Consider the separate propositions that represent the three ideas in the sentence "A net force on a body causes acceleration." Note that this statement contains three ideas:

1) A force on a body
2) A force causes acceleration
3) Net force

The respective propositions may be stated:

1) Relation - on; arguments - force (subject, S), body (object, O)
2) Relation - causes; arguments - force (subject, S), acceleration (object, O)
3) Relation - net; argument - force (object, S)

Figure 5A depicts the separate propositions in node-link form. The nodes, shown as circles, represent the propositions. Each link, shown as an arrow, is directed toward an element of the proposition and is labeled to identify the function of the element: R for relation, S for subject, and O for object. Propositions that contain common elements may be interconnected to form propositional networks. The three separate propositions in Figure 5A may be grouped about the common element "force" and thus constitute one propositional network (see Figure 5B). This relatively simple network illustrates a possible structure of declarative knowledge in LTM.

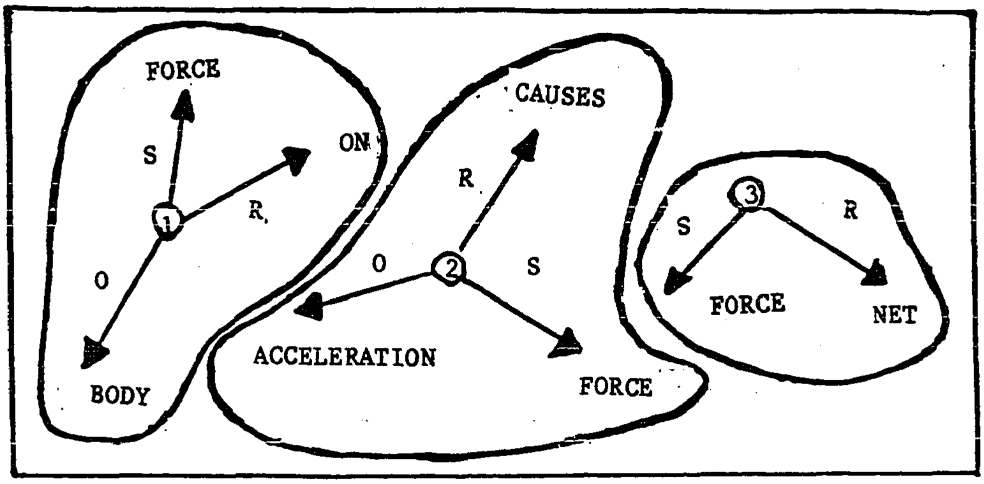


Figure 5A. Separate propositions

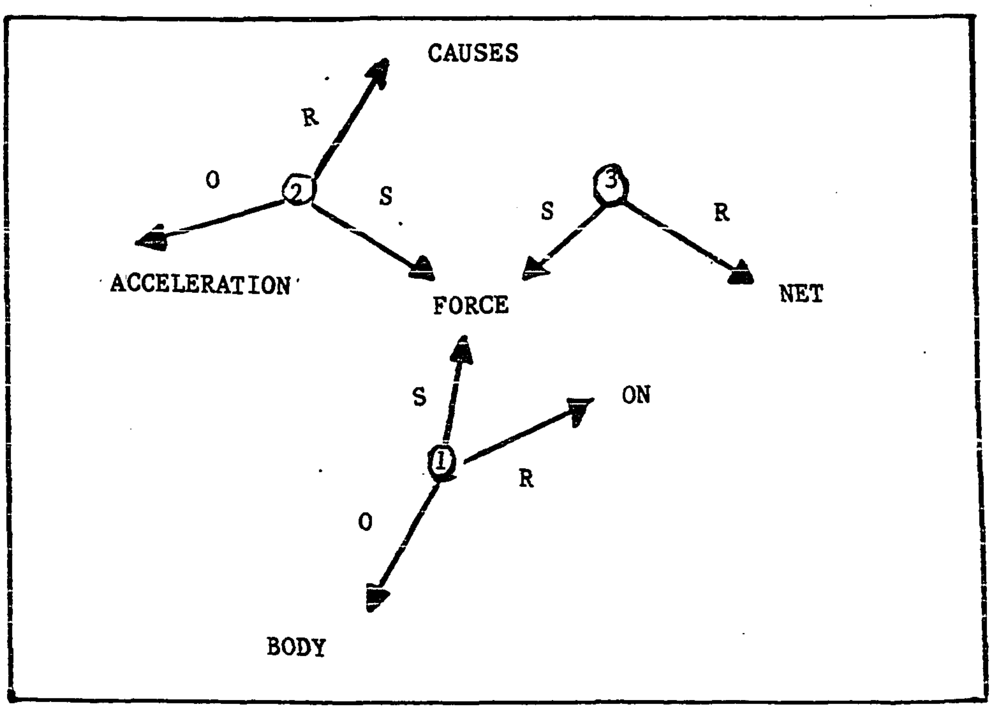


Figure 5B. Propositional network

Knowing that a net force on a body causes the acceleration of the body may not enable a person to solve a problem. Consider the physics problem:

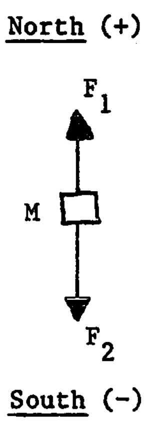
> Two forces, F(1) in a northerly direction and F(2) in a southerly direction, act simultaneously on a body of mass m that is free to move with negligible friction. F(1) is greater than F(2). Calculate the acceleration of the body.

A solution, shown in Figure 6A, results in the acceleration, a = [F(1) - F(2)]/m, in a northerly direction. Note the direction of the acceleration is indicated by an ↑. A problem solver, carrying out this solution, must use declarative knowledge as well as procedural knowledge. Procedural knowledge can be represented by means of productions. A production is a condition-action statement. Consider a sample production:

> IF The name on the envelope is P. P. Solver
> And the label on the mailbox indicates
> P. P. Solver
> THEN Place the envelope in the mailbox.

The IF clause is the condition statement and the THEN clause is the action statement. Productions may be joined to form systems or networks. Note that, in Figure 6B, the action of a production becomes the condition for the succeeding production, e.g., the action of P(1) becomes the condition for P(2) and the action of P(2) becomes the condition for P(3). Productions, in this fashion, may be interconnected through the "flow of control" (Gagne, 1985).

Acceleration is a vector quantity. Such quantities include magnitude and direction. The production system in Figure 6B results in the magnitude of the acceleration equaling [F(1) - F(2)]/m. The



$$F = Ma$$

$$F_1 + (-F_2) = Ma$$

$$F_1 - F_2 = Ma$$

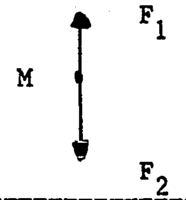
$$a = \frac{F_1 - F_2}{M}$$

Figure 6A. Sample solution to a mechanics problem

P_1 — P_1 Result

IF The goal is to calculate the acceleration $\underline{a}$ of the body and M, F_1, and F_2 are known

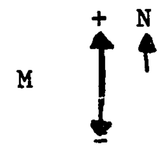
THEN Draw a diagram indicating each force, the mass, and the geographical orientation <u>and</u> set subgoal to calculate the net force on the body.



P_2 — P_2 Result

IF Subgoal is to calculate the net force on the body.

THEN Assign "positive" to North and "negative" to South <u>and</u> calculate the net force by adding F_1 <u>and</u> F_2 write the net force in terms of F_1 and F_2 and set subgoal to write an equation that includes net force, mass, and acceleration $\underline{a}$.



$\Sigma F = F_1 + (-F_2)$

$\Sigma F = F_1 - F_2$

P_3 — P_3 Result

IF Subgoal is to write an equation that relates net force, mass, and acceleration $\underline{a}$.

THEN Write the equation <u>and</u> set subgoal to substitute the net force in terms of F_1 and F_2 into the equation.

$\Sigma F = ma$

P_4 — P_4 Result

IF Subgoal is to substitute the net force in terms of F_1 and F_2 into the equation.

THEN Substitute the net force in terms of F_1 and F_2 into the equation <u>and</u> write the resulting equation <u>and</u> set subgoal to solve it for $\underline{a}$.

$F_1 - F_2 = ma$

P_5 — P_5 Result

IF Subgoal is to solve the resulting equation for $\underline{a}$

THEN Solve the resulting equation for $\underline{a}$ and write the answer.

$$a = \frac{F_1 - F_2}{m}$$

Figure 6B. Production system for a mechanics problem direction can be deduced from knowing that $\Sigma\vec{F} = m\vec{a}$ where $\vec{F}$ and $\vec{a}$ are vector quantities. In effect, $\Sigma F = ma$ is a constrained form of $\Sigma\vec{F} = m\vec{a}$ that is to be applied to only one spatial dimension. Implied in $\Sigma\vec{F} = m\vec{a}$ is that $\Sigma\vec{F}$ and $\vec{a}$ have the same direction. $\Sigma F = F(1) - F(2)$ acts in a positive or northerly direction. Knowing that $\Sigma\vec{F}$ and $\vec{a}$ are in the same direction may be viewed as an idea for which a proposition could be written, i.e., it is known in declarative fashion. Figure 6B, in which the production system displays procedural knowledge, includes arrows of different lengths. The declarative knowledge that arrows are used to represent vector quantities with the arrow heads indicating direction and the lengths of the arrows being proportional to the magnitudes of the represented quantities is necessary to bring about the result of P(1).

When the distinction between declarative and procedural knowledge is made, it seems evident that the solution of physics problems involves both kinds of knowledge. The distinction between the two kinds of knowledge, often useful, is not necessarily sharp. Consider, for example, P(3): The subgoal is to write an equation that relates net force, mass, and acceleration $\underset{\sim}{a}$. It seems tenable that a problem solver may have stored the knowledge of this equation in production form:

IF The mass of an object remains constant and a net force acts on the object
THEN The net force is proportional to the acceleration

and

IF The net force on an object is constant

THEN The acceleration of the object is inversely
proportional to its mass

and

IF A net force acts on an object
THEN The resulting acceleration is in the same
direction as the net force

When, in the course of working out the solution to a problem, it becomes necessary to incorporate these ideas in one mathematical relation, the problem solver may write $\Sigma\vec{F} = m\vec{a}$. The IF clause of P(3), for example, may access these productions. It seems equally reasonable that the knowledge of this equation may have been stored as a propositional network containing the propositions:

1) Force is proportional to acceleration
   (as mass is constant)
2) Net force
3) Mass is inversely proportional to acceleration
   (as force is constant)

which may be written as:

1) Relation - proportional to; arguments -
   force (S), acceleration (O)
2) Relation - net; argument - force (S)
3) Relation - inversely proportional; arguments -
   mass (S), acceleration (O)

Figures 7A and 7B contain the separate propositions and the network. This network may be accessed by the IF clause of the same production, P(3), in the problem under consideration.

The distinction between declarative and procedural knowledge reflects the more or less static nature of declarative knowledge and the relatively dynamic character of procedural knowledge. Declarative knowledge supplies information that is transformed by activated procedural knowledge (Gagne, 1985). Rumelhart and Norman (1978) do not

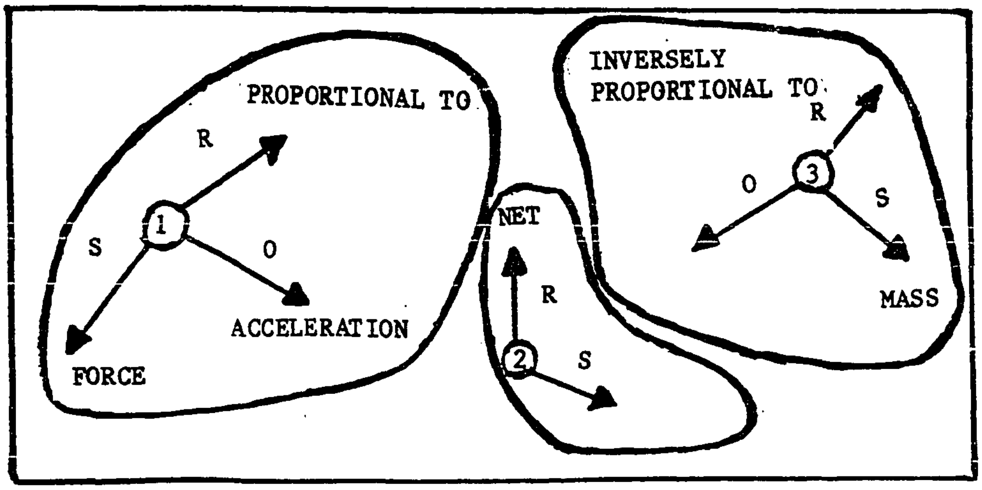


Figure 7A. Separate propositions

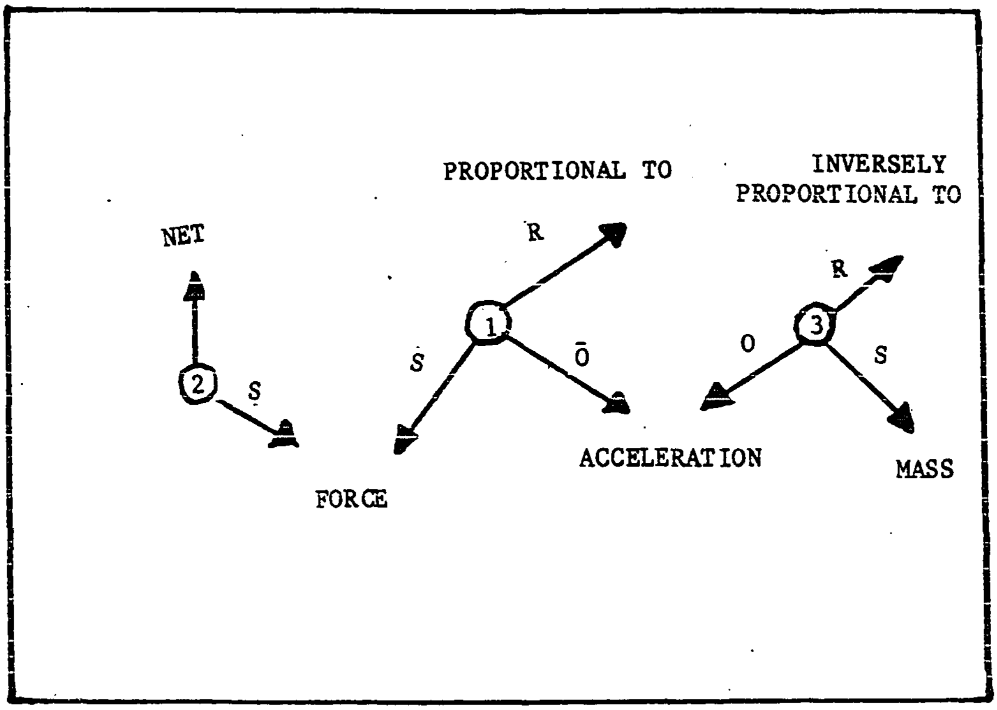


Figure 7B. Propositional network

believe the distinction to be useful. The distinction is fundamental to the ACT (to act or to accomplish a task) system (Anderson, 1982), a skill acquisition model, in which facts are encoded in a propositional network and productions serve to encode procedures. The ACT system (a computer simulation) and the underlying theory include an interpretative system that describes the behavior of a novice in a new domain. The ACT theory claims that initial knowledge in a new domain exists in declarative form as the ACT system gradually changes this declarative knowledge to a procedural form. The ACT theory, in explaining how a cognitive skill may be acquired, involves the skill being transformed from the declarative stage to the procedural stage of development.

A knowledge structure that includes both declarative and procedural knowledge is the schema. As viewed in this study, a schema is an organized structure within memory. Greeno (1973) believes schemata to be propositional (concepts from general experiences) and algorithmic (rules that operate on concepts). Consonant with this meaning, Gagne (1985) includes static qualities (structures), active qualities (expectancy toward information), conscious use (for example, retrieval guidance), and automatic use (for example, recognition of a new instance of a concept). Andre (1986) operationally understands schemata to include the representation of concepts (categories), principles or rules (relationships among concepts), and skills (activities that require several steps). The notion of a schema, frequently used in cognitive work, is not new: Bartlett (1932) believed a schema to be a structure that refers to an active organization of past reactions.

It is evident that differences in the meanings assigned to the term "schema" exist but Mayer (1983) finds a generally applicable meaning to include four common elements:

> General - a schema may be used in a wide variety of situations as a framework for understanding information
> Knowledge - a schema exists in memory as something that a person knows
> Structure - a schema is organized around some theme
> Comprehension - a schema contains "slots" that are filled in by specific information in the passage (p. 209).

Mayer (1983) concludes with:

> Thus, a schema is a generalized knowledge structure used in comprehension. A schema serves to select and organize incoming information into an integrated, meaningful framework (p. 209).

A general consensus on the use of "schemata" does not exist. Mayer (1983) quotes Scriven (1977):

> I do not think that talking about 'schema' or 'schemata' or 'frameworks' does much that 'tuning' does not or that 'gestalt' did not (p. 234).

Andre (1986), on the other hand, holds that the formalism of a production system can be used, in a unifying manner, to represent Gagne's (1977) intellectual skills and other schemata.

Even as a consensus on the use of "schemata" does not exist, the empirical evidence for their existence and the accompanying use of the term have made them part of the fabric of problem-solving research within the realm of the information-processing approach.

Cognitive psychologists thus acknowledge schemata which are used in cognitive knowledge theory as it deals with the ways in which knowledge is stored in memory. Representations are particular schemata by means of which problem solvers describe the environment and solve problems by mental operations on these descriptions. Representations are important to this study and are considered in the next section.

Problem Representation

A person who wants to achieve a goal without immediately knowing how to arrive at it is involved in a problem (Newell & Simon, 1972; Gagne, 1985). Mayer (1983) claims that a definition of "problem" should include that "1) the problem is presently in some state, but 2) it is desired that it be in another state, and 3) there is no direct, obvious way to accomplish the change" (p. 5). Clearly, this definition is parallel to the description of a problem as included earlier in this study:

> To have a problem implies (at least) that certain information is given to the problem solver: Information about what is desired, under what conditions, by means of what tools and operations, starting with what initial information, and with access to what resources (Newell & Simon, 1972, p. 73).

Problem solving is a process in which the "problem space" is searched. The problem space is the set of possibilities for a solution as perceived by a problem solver (Newell & Simon, 1972). Mayer (1983) views the space as the internal representation of the initial state, the goal state, the intermediate states, and the operators (moves that change one state to the next); Gagne (1985) considers the space to be the set of all solution paths that can lead to the goal.

Newell and Simon (1972), in their general problem-solving model, outline an organization of the problem-solving process. As input from the environment is translated (encoded), an internal representation of the external environment is formed. The problem solver proceeds in the framework of that representation; it is this representation that may "render the problem solutions obvious, obscure, or perhaps unattainable" (p. 88).

The problem solver now responds to the representation by the selection of a particular problem-solving method which is formulated and interpreted in terms of the internal representation. This method, as it is being applied, controls the internal and external behavior of the problem solver. When the application of the method ceases, a solution may have been formed, another method may be selected, the problem may be reformulated in terms of a different internal representation, or the solution process may be abandoned. Subgoals may result from the application of problem-solving methods during the search of the problem space.

The Gestalt psychologists viewed problem solving as a search in which aspects of problems were related to each other. The problem solver strives for structural understanding which is the ability to comprehend how all the parts of the problem can be arranged to satisfy the requirements of the goal. The elements of the problem are reorganized as the problem is being solved. Consider the problem of being given six sticks, equal in length, and being asked to form four equilateral triangles with each side being one stick long. Figure 8A depicts an incorrect solution as each triangle contains a ninety-degree angle and thus is not equilateral. This two-dimensional solution may be changed to a three-dimensional arrangement: A pyramid with one triangle as the base and the three remaining triangles as sides may be formed as shown in Figure 8B. The Gestaltists label a new way of looking at a problem as insight. Changing the representation from two-dimensional to three-dimensional allowed for the formation of a solution.

Mayer (1983) uses a problem based on the work of Judson and Cofer (1956). Consider the sequence SKYSCRAPER CATHEDRAL TEMPLE PRAYER and select the word that does not belong. Next, select the word that does not belong in the sequence CATHEDRAL PRAYER TEMPLE SKYSCRAPER. Judson and Cofer (1956) found that subjects generally chose PRAYER in the first sequence and selected SKYSCRAPER in the second series. One explanation is that problem solving requires assimilation of the elements of the problem into the past experience of the problem solver. This type of interpretation is consistent with meaning theory. The meaning theorists hold that the restructuring or reorganizing process as emphasized by the Gestalt psychologists needs to be guided by relationships between the schemata in the problem solver's memory and the elements of the problem under consideration. The Gestalt theory emphasizes the internal structure of the problem while the meaning theory includes relationships to schemata that exist in the memory of the problem solver. The relationships that link the existing schemata with the problem contribute to the problem representation.

An interesting problem, attributed to Duncker (1945), that

Consider the problem:

Given six sticks, arrange them to form four triangles that are equilateral and with each side one stick long.

The solution: Some subjects take the six sticks and form a square with an X in it:

Each triangle has a 90 degree angle and thus is not equilateral.

(Mayer, 1983)

Figure 8A. Incorrect solution of a sample problem

A hint: Change from two to three dimensions.

A pyramid with one triangle as the base and the three remaining triangles as sides may be formed.

Insight: A new way of looking at a problem, such as changing from two to three dimensions.

(Mayer, 1983)

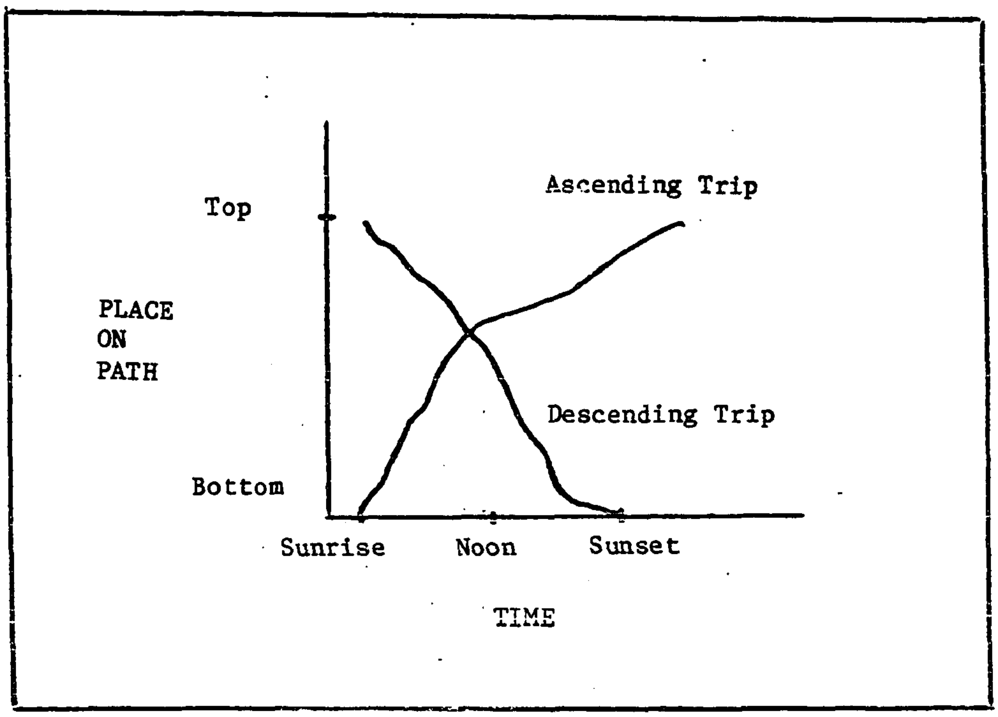
Figure 8B. Role of insight in correct solution of a sample problem illustrates the importance of problem representation is used by Mayer (1983):

> A monk began to climb a mountain at sunrise. He reached the temple at the top as the sun was setting and meditated all night. At sunrise of the next day, he came down the mountain, following the same path, but moving at a faster rate, of course. When he reached the bottom he proclaimed: "There is one spot along this path that I passed at exactly the same time of day on my way up the mountain as on my way down." Can you prove that the monk is correct? (pp. 75, 76).

This problem may be difficult to solve algebraically. Mayer (1983) suggests that the problem be visualized, i.e., a representation be formed, as shown in Figure 9. It can be seen that there must be a point at which the time of day is the same for the ascending and descending trips. Another representation is helpful: Picture two monks, one at the bottom going up and one at the top going down and ask whether they will be at the same place at the same time at some point during their respective journeys (Mayer, 1983).

Before continuing the consideration of problem representation, a brief view of the methods of computer simulation and think-aloud protocols is in order.

Computer simulation models as used in much problem-solving research are able to solve the kind of problems solved by human problem solvers. Such models usually contain a STM with a capacity that approximates that of STM in the human problem solver. LTM in such models consist of collections of productions. The models test the condition sides of these productions by searching STM (working memory) for the presence of information elements that match these condition sides. If matches are



(Mayer, 1983)

Figure 9. A problem representation

found, the production is implemented and information is taken from or added to STM. An intermediate memory may be included which serves to hold information transferred from STM which, at appropriate times, may be returned to STM. Such memory does what a piece of paper accomplishes for a human problem solver. Models that approximate the behavior of humans with varying degrees of expertise yield results that may be compared to the results obtained by human problem solvers.

Such comparisons are often made by the use of think-aloud protocols. As human problem solvers solve problems, they "think aloud" as their comments are being recorded on tape. The verbatim tape transcripts are then edited and analyzed. The quantitative statements are listed and sequenced. Such sequences are then compared to the results from computer simulations.

Larkin, McDermott, Simon, and Simon (1980) describe two computer-implemented models that solve problems in ways that reflect the solutions of more- and less-competent human problem solvers. The models differ in the strategies that are employed in the selection of physics principles. The means-ends model focuses on the goal quantity, writes an equation that contains that quantity, and then works backward in order to find equations that contain quantities that still remain unknown. In means-ends analysis the program or a human problem solver works on one subgoal at a time: The difference between the present state of knowledge about the problem and the state that is necessary for the solution of the problem is assessed. The knowledge-development model uses patterns of information in the development of new information. The models also differ in the degree of automation. The means-ends model writes an equation after which variables in this equation are connected to variables in the problem statement by other individual equations. The knowledge-development model, on the other hand, combines the selection and application of a principle, in effect developing new information in a single step.

The computer results were matched with the data from one novice and one expert in a study by Simon and Simon (1978) where they worked 19 problems in linear kinematics; in-depth information on the techniques used by these subjects was obtained. A further match was made with the results from 11 experts and 11 novices who solved two dynamics problems; this study resulted in a broader view of how experts and novices solve problems. The computer results characterize the sequences of applied principles by novices and experts: Novices set many subgoals as they use means-ends analysis, and experts tend to set fewer goals as they use more efficient and domain-dependent methods. The results also parallel the automation displayed by experts which differs from the explicit linking of variables to information in the problem statement as manifested by novices. Computer simulations are viewed as effective tools for research in epistemological studies.

The problems are submitted to the programs in list form in which the language statements of the problems have been encoded: These representations are thus given to the program rather than constructed by the program after reading the problem statements.

Representations need not necessarily be given to such programs.

Novak (1977) describes a computer program that solves physics problems stated in English. The English sentences in the problem statement are transformed into a semantic network form that includes a language-free internal model of the objects and their characteristics and interrelationships, a geometric model, a set of equations, a picture model, and a set of canonical object frames which interpret the actual objects as canonical objects. A canonical object frame - Minsky (1975) introduced the term "frame" - is a schema that abstracts features of actual objects which allows for the definition of physical laws in terms of the canonical objects which approximate the behavior of real objects. The context of a real object determines which canonical frame is to be used. Novak (1977) uses the example of a person being modeled as a point mass when sitting on a plank but as a pivot when carrying this plank. A specialist program operates on the canonical representation and the subsequent solution is applied to the specific object representation in order to provide a specific solution.

Problems considered to be difficult can be solved by some computer programs that, surprisingly, cannot solve simpler problems. The importance of problem representations to subsequent solutions is evident as deKleer (1977) describes NEWTON, a computer program that employs multiple representation in solving mechanics problems. NEWTON allows simple questions to be answered directly and has the capacity for producing plans that can solve more complex problems: Qualitative knowledge is used to approximate expected behavior of a physical system by means of a process called "envisioning," while the quantitative knowledge built into the program is organized in groups of mathematical equations which are accessed when a problem cannot be solved qualitatively. NEWTON, while not a general problem solver, generates appropriate representations that allow for the solution of simple and complex problems.

Simon and Simon (1978) asked an expert subject with a strong mathematical background and wide experience in solving problems of the specific task type and a novice subject with adequate algebraic background and completion of a single physics course to solve 25 kinematics problems from a high school physics text. Part of the purpose of the study was to describe explicit knowledge of physical laws that students must have and the ways in which such knowledge must be organized as a necessary underpinning for problem solving. The "thinking-aloud" protocols of the subjects allowed for the conclusion that the expert used physical intuition in solving the problems. Physical intuition, in this context, is interpreted by Simon and Simon (1978):

> When a physical situation is described in words, a person may construct a perspicuous representation of that situation in memory. By perspicuous representation, we mean one that represents explicitly the main direct connections, especially causal connections, of the components of the situation (p. 337).

The offered interpretation, with a disclaimer of the evidence being totally decisive, is that the expert translated the English statements of the problems into physical representations and subsequently "used those representations to select and instantiate the appropriate equations" (Simon & Simon, 1978, p. 337), after which he solved the problems. Computer simulation comparisons indicated that the protocol of the novice might be called "algebraic" in that she appears to move directly from problem statements to equations without passing through the representation phase as did the expert whose mode of problem solving might be called "physical."

Larkin (1980) examined the relationships between problem representation and subsequent solution in a study in which she asked six experienced physics problem solvers to solve a difficult physics problem that lends itself to being solved in many different ways. The five subjects who made reasonable progress constructed one or more qualitative representations which were used to test the feasibility of various theoretical approaches. The approach is then discarded or the construction of equations begins. The construction of a representation is viewed to be central to problem solving:

> In no case was the theoretical approach changed after the solver began to construct equations. Thus, qualitative theoretical representations would seem to be crucial in the important task of selecting an approach (Larkin, 1980, p. 122).

Larkin (1980) mentions that she and John McDermott are developing programs that simulate the order in which expert and novice subjects apply physics principles as they solve mechanics problems. They believe that the use of the qualitative theoretical representation will constitute the major difference between the expert and novice programs.

Heller and Reif (1984) formulated a theoretical model that specifies the knowledge and procedures necessary for human problem solvers to generate good initial representations (or "descriptions") of scientific problems. These representations describe problem statements in terms of specific concepts in the domain-specific knowledge base for mechanics problems. The subjects in their study to test the model were 24 paid undergraduates enrolled in the second course of an introductory physics sequence at the University of California at Berkeley. The subjects had studied mechanics principles and solved problems of the kinds used in the study. The subjects were randomly selected from those volunteers who finished their previous physics course with a grade of B- or better. The subjects were randomly assigned to three groups of eight subjects each. The subjects solved three approximately matching pairs of physics text problems. The pairs of problems were divided into two approximately equivalent sets. Half of the subjects in each group completed one set as a pretest and the second set during the experimental treatment. The other half of the subjects completed these sets in the opposite order.

The subjects were given a printed summary of the mechanics principles relevant to the problem sets, which they were asked to read and use for reference at any time. The pretest was completed without external guidance. The subjects then completed a practice session during which they solved mechanics problems under external control as verbal directions were read to the subjects. Each direction had to be performed by the subjects before listening to the next direction.

The subjects in the first treatment group completed the problem set under external guidance that implemented the formulated model. The second treatment group solved the problems under external guidance that implemented an alternate model that approximately simulates the descriptive advice frequently found in physics texts and is less inclusive and explicit than the formulated theoretical model. The third experimental group constituted the control group and solved the problems without external guidance.

According to Heller and Reif (1984), the performance of the subjects on the pretest and the performance of the control group indicated insufficient knowledge for solving the problems in an adequate fashion. Many students, in spite of having received good grades in a physics course in which they did formal work in mechanics, generated incomplete representations of the problems that led to incorrect solutions. The formulated model allowed the subjects to construct explicit and correct representations that markedly facilitated correct solutions.

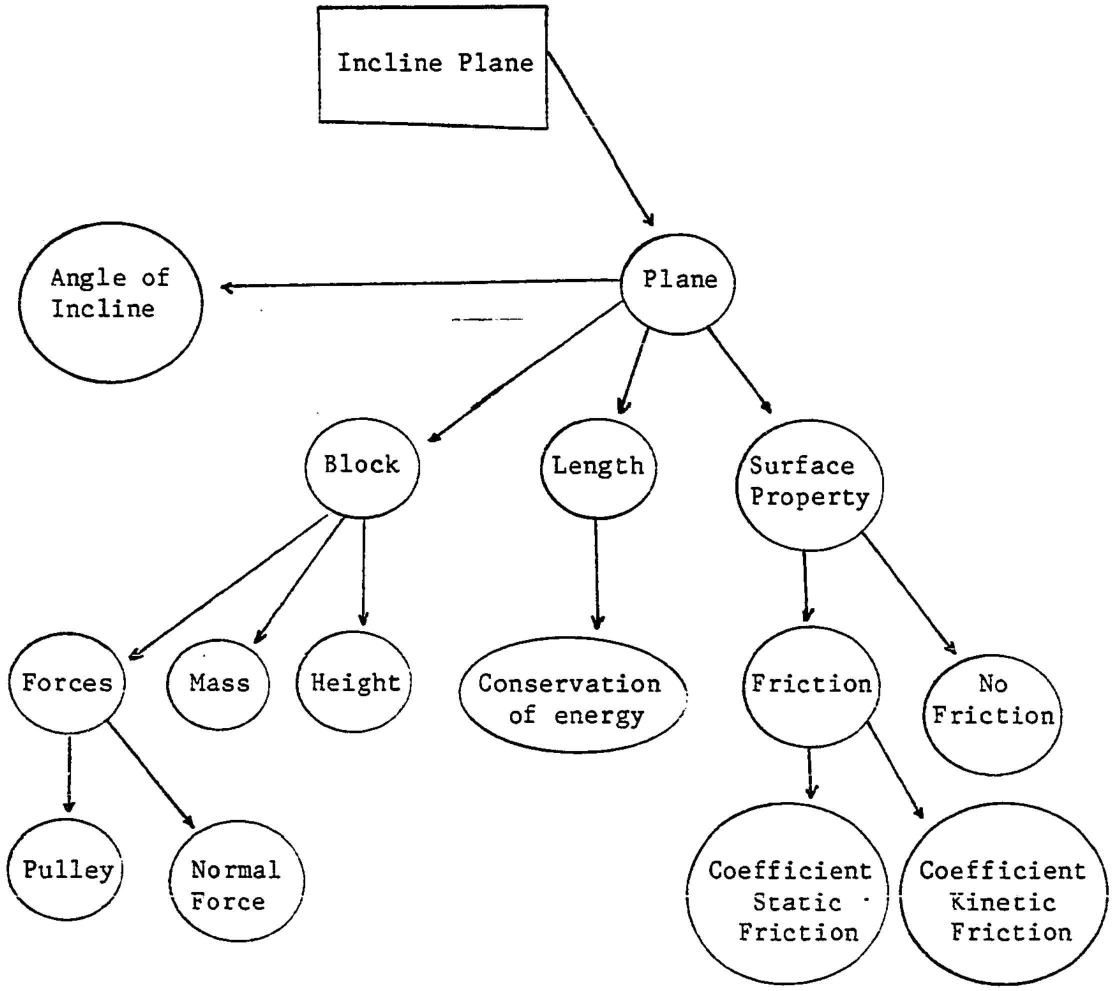
The work of Chi, Feltovich, and Glaser (1981) includes a series of studies that examines differences in representations of physics (mechanics) problems that are constructed by novices and experts. The guiding hypothesis is that:

> ...the representation is constructed in the context of the knowledge available for a particular type of problem. The knowledge useful for a particular problem is indexed when a given physics problem is categorized as a specific type (p. 122).

The subjects in the first study were eight advanced Ph.D. students (experts) and eight undergraduates (novices) who had completed one categories and their associated knowledge in the domain-specific knowledge base constitute the schemata that determine the quality of the representation process and subsequently infer that the problem schemata of novices and experts are different because their categorization processes are different.

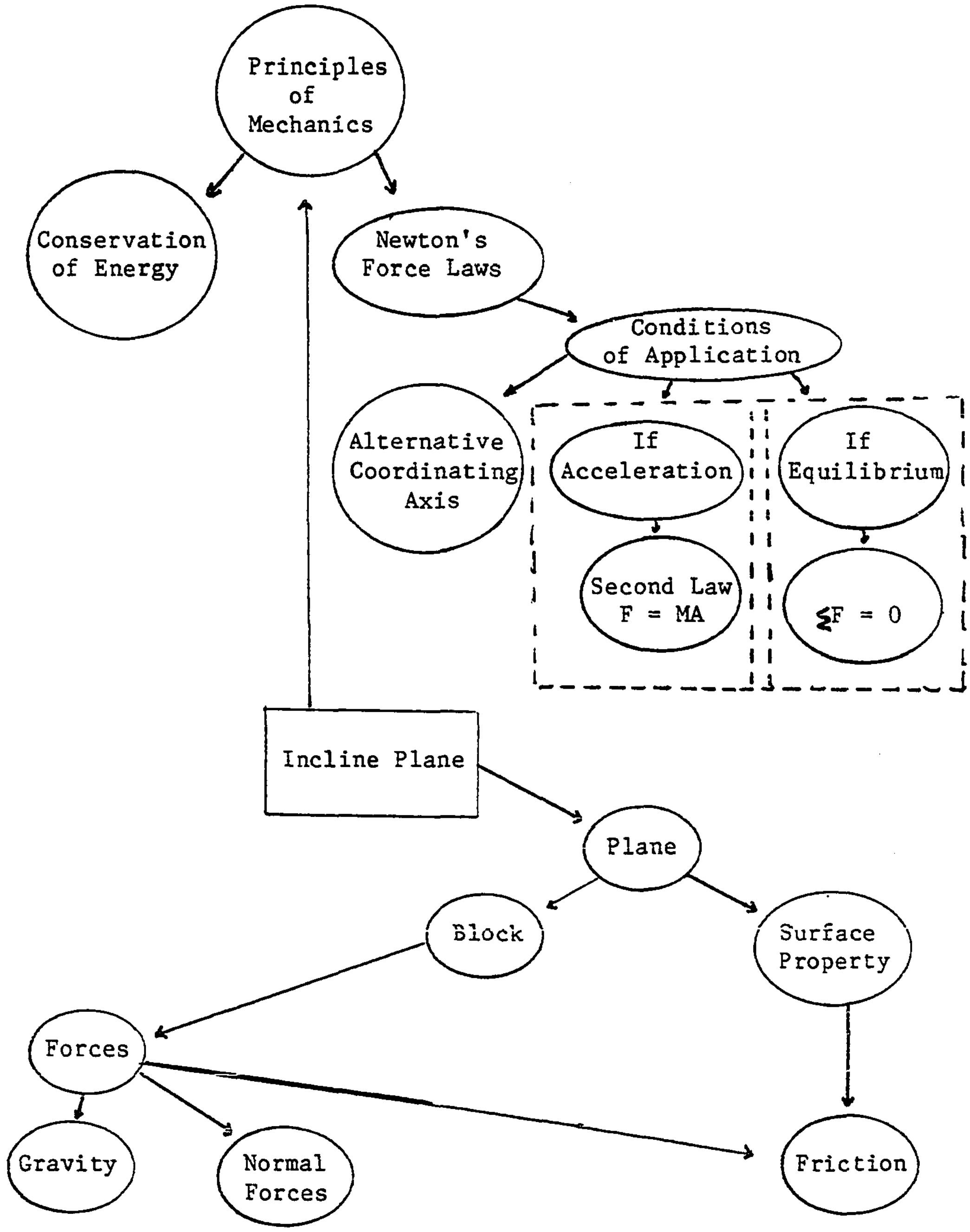
In the third study by Chi et al., assuming that the category descriptions generated by the subjects represent labels that are used to access particular schemata, a collection of 20 category labels was given to two experts and two novices. The subjects were asked to state everything they knew about problems involving the category labels. The results, depicted in Figures 10 and 11 in node-link form, show that surface (structural) features are shared by novice subjects (Figure 10) and expert subjects (Figure 11) but that the expert subjects possess principles and the conditions for their applicability as shown in the top of Figure 11.

Two expert subjects and two novice subjects, different from the subjects in the second study, were given the set of counterbalanced problems used in the second study. The subjects were asked to think out loud about the "basic approaches" that they would use for solutions and were asked to state explicitly the "basic approaches" and the problem features that caused their choices. Analysis of the protocols showed that the expert subjects used the same terms for their "basic approaches" (major principles) as other experts did in the sorting tasks. The novice protocols were impossible to analyze as they lacked explicit statements and contained only very general statements. The



(Chi, Feltovitch, & Glaser, 1981)

Figure 10. Novice schema associated with an inclined plane



(Chi, Feltovitch, & Glaser, 1981)

Figure 11. Expert schema associated with an inclined plane

semester of mechanics. The subjects were asked to categorize 24 mechanics problems in any way they desired. The subjects also gave brief reasons (descriptions) for having chosen their respective categories.

A cluster analysis revealed that the novices classified according to surface features, i.e., objects such as levers, springs, and pulleys; specific terms such as friction and force; spatial arrangement of the components of the problem statements, and that the experts classified according to deep structures, i.e., physics laws (for example, conservation of linear momentum) and concepts (for example, angular speed).

The second study was done for the purpose of testing the findings from the first study. A set of 20 problems in which surface features and deep structures were counterbalanced, i.e., each problem contained one deep structure and one surface feature, were sorted. Chi et al. (1981) expected "that novices would group together problems that have the same surface structures, regardless of deep structure, and experts would group together those problems with similar deep structures, regardless of the surface structures. Individuals of intermediate competence should exhibit some characteristics of each."

The results show that these expectations were realized and that advanced novices begin to use principles and laws rather than to rely only on surface features in their categorizations as the results of a novice, an advanced novice (a fourth-year undergraduate physics major), and two experts are reported. Chi et al. (1981) believe that the expert protocols included descriptions of the problem states and conditions of the physical situations described by the problems. Some of these were transformed or derived such as "no external forces," called "second-order" features as they do not appear in literal form in the problem statements. Chi et al. (1981) believe that literal parts of the problem statements are changed into "second-order" features that activate category schemata for certain problem types. These schemata are organized by physics laws and direct the completion of the problem representations and initiate mathematical solutions. Expert subjects in a fourth study were seen to make qualitative analysis prior to working with equations, a behavior also observed in the studies by Simon and Simon (1978) and Larkin (1980).

The categorization process seems to be a promising technique for the study of representation of physics problems that allows for inference of schemata that exist in the domain-specific knowledge base.

CHAPTER III. METHODOLOGY

Description of Subjects and Tasks

The total novice sample consists of 94 students in Physics 221, Introduction to Classical Physics (spring term of 1985-1986), at Iowa State University. The students had completed the mechanics portion of that course at the time of task engagement. The students had not used a version of Fundamentals of Physics, the text from which the problems in Problem Set 1 and some of the problems in the other three problem sets were selected. The sample was categorized according to ACT science scores: Group 1 - ACT less than or equal to 27, Group 2 - ACT greater than or equal to 28 but less than or equal to 32, and Group 3 - ACT greater than or equal to 33. These groups are used in the post-hoc analysis investigating possible differences among novices with respect to the outcomes of the sorting tasks. The range of each group reflects the division of the subjects into a group that might be expected to include relatively poorer problem solvers, a middle group, and a group that might be expected to include the relatively better problem solvers. The assignment of the subjects in each group to one of four problem sets was random.

The intermediate (I) sample consists of five students who had completed Physics 361, Classical Mechanics, at Iowa State University (the 1985-1986 academic year) prior to the time of task completion. This sample is involved only with Task 3, used in the testing of the third hypothesis.

The total expert sample consists of 20 physicists who hold the Ph.D. degree in physics. All but two of these have taught an introductory course in calculus-based physics. Fourteen of the total sample have such a course as a part of their usual teaching load and 16 have used a version of Fundamentals of Physics. The average number of years of experience in teaching courses such as Physics 221 approximates 12 years. Five experts, as they became available, were assigned to each problem set.

Four problem sets were used. Each problem in each set is stated completely and is cited in a separate appendix. Problem References A, B, C, and D, respectively, contain Problem Sets 1, 2, 3, and 4. Each problem was typed on a 5-inch x 8-inch card. Each task consisted of a. sorting a problem set and b. solving one of the sorted problems. There was no overlapping of subjects. Each subject sorted only one problem set and solved only one problem in that sorted set. Figure 12 shows the relationships among tasks, hypotheses, and problem sets. The numbers of subjects assigned to each task are also included.

The directions that were given to the subjects are:

> Please read all of the directions before beginning the tasks.
>
> You have been given a set of problems (a stack of notecards) bound with a rubber band.
>
> Task A - Please sort these problems according to any similarities (or patterns of similarities) that you perceive among them. After removing the rubber band, sort the problems by making one subset (a smaller stack) of problems based on each similarity (or pattern of similarities) that you perceive. Each smaller stack may consist of any number of cards. After you have finished

TASK	HYPO THESIS	PROBLEM SET	NUMBER OF PROB- LEMS IN SET	NOVICE SUBJECTS	EXPERT SUBJECTS	INTER- MEDIATE SUBJECTS	N
1	1	1	24	28	5	0	33
2	2	2	24	20	5	0	25
3	3	3	16	23	5	5	33
4	4E	4	16	23	5	0	28
	4N						
				--	--	--	--
				94	20	5	119

Figure 12. Table: Summary of tasks, hypotheses, problem sets, and subjects

> the sorting process and have a number of smaller stacks in front of you, take a blank notecard from the table and place one such blank notecard on top of each of the smaller stacks. On each of these cards write a COMPLETE explanation for having placed (sorted) the problems in that particular subset. Please be very complete and explicit. Simple sketches or diagrams used to add to your written statements are valuable. You are encouraged to use such sketches or diagrams. When finished, place a rubber band around each stack. Then place all your stacks on top of each other, add the card with your name to the top of the stack and bind with two criss-crossing rubber bands.
>
> Task B - At the bottom of your problem set is a stapled, folded sheet. Please do not remove the staple until you have finished Task A totally. After removing the staple, solve the problem, writing all steps and jotting down related thoughts in a very complete mannner. Cross out errors and otherwise make changes in a manner allowing all your writing to remain visible.
>
> When finished, please give the bound stack of cards and the separate solution to me.

It was made clear to the subjects that the subsets might be altered any number of times as Task A was being completed.

Collectively, the numbers on the cards (numbers of the problems) in each sorted subset determine the composition of the categories. The written rationales on the cover cards of the subsets determine the labels of the categories. Comments such as "deal mainly and almost entirely with friction" and "had to know the conservation of momentum law to find velocity" were retained and collated. Comments such as "I don't know how to solve this" and "looks difficult" were excluded.

The problem sets are now described as related to the hypotheses.

Hypothesis 1 - Experts will categorize physics (mechanics) problems on basis of deep structures and novices will categorize these problems on basis of surface features.

The work on Hypothesis 1 is designed to replicate part of the findings of the Chi et al. study (as stated by this hypothesis) and, simultaneously, is used to validate the dendogram methodology.

Problem Set 1 consists of 24 mechanics problems selected from Chapter 5 through 8 in Fundamentals of Physics by Halliday and Resnick (1974). Twenty-two of these problems are identical to 22 problems in a 24-problem set used by Chi et al. (1981).

The novice sample consists of 28 Physics 221 students and the expert sample includes five Ph.D. physicists.

Hypothesis 2 - Experts will categorize a different set of physics (mechanics) problems on basis of deep structures and novices will categorize this set on basis of surface features.

The purpose of Hypothesis 2 is to attempt a degree of generalization for the results of the testing of Hypothesis 1, i.e., to determine whether the Hypothesis 1 effects also exist for a different problem set.

Problem Set 2 consists of 24 mechanics problems selected from physics texts of a level similar to that of Fundamentals of Physics. A few of these 24 problems are selected from Fundamentals of Physics.

The novice sample consists of 20 Physics 221 students and the expert sample includes 5 Ph.D. physicists.

Hypothesis 3 - Experts will categorize physics (mechanics) problems according to deep structures regardless of surface features and novices

<u>will categorize these problems according to surface features regardless of deep structures. Intermediates will reveal a categorizing pattern that is characterized by a mixture of deep structures and surface features.</u>

Problem Set 3 is of basic importance to this study. The results of Task 3 attained by the expert subjects serve as the basis of comparisons among other results as explained in the section entitled <u>Treatment of Data</u>.

Problem Set 3 consists of 16 mechanics problems in which each problem contains one surface feature and one deep structure. The four deep structures (D) in the set are Newton's Second Law (D1), Conservation of Energy (D2), Conservation of Linear Momentum (D3), and Conservation of Angular Momentum (D4). The four surface features (S) are the Spring, (S1), the Inclined Plane (S2), the Pulley (S3), and Terms (S4). Terms (S4) includes physical arrangements of objects and literal physics terms in text of the problems. The problem set may be shown in matrix form:

	D1	D2	D3	D4
S1	S1D1	S1D2	S1D3	S1D4
S2	S2D1	S2D2	S2D3	S2D4

S3| S3D1 S3D2 S3D3 S3D4

S4| S4D1 S4D2 S4D3 S4D4

Clearly, the set contains four deep structures and four surface features.

Problem 3.2, the second problem in Problem Set 3 (Appendix C), serves as a specific example:

> The force required to compress a horizontal spring an amount x is given by F = ax + b(x to the 3rd power) where a and b are constants. If the spring is compressed an amount l, what speed will it give to a ball of mass M held against it and released? (Giancoli, 1984, p. 113).

The surface feature is the Spring (S1) and the deep structure is the Conservation of Energy (D2). The problem is then designated as S1D2 in the matrix.

The novice sample consists of 23 Physics 221 students, the intermediate sample consists of five students who had completed Physics 361, and the expert sample includes five Ph.D. physicists.

Hypotheses 4E and 4N are considered together.

Hypothesis 4E - Experts will categorize a set of physics (mechanics) problems according to deep structures regardless of surface features with the number of established categories approximately equal to the number of deep structures contained in the set.

Hypothesis 4N - Novices will categorize a set of physics (mechanics) problems according to surface features regardless of deep structures with the number of established categories approximately equal

<u>to the number of surface features contained in the set</u>.

Problem Set 4, used for testing Hypotheses 4E and 4N, consists of 16 mechanics problems in which each problem contains one deep structure and one surface feature (Task 4). The two deep structures (D) in the set are Newton's Second Law (D1) and conservation of energy (D2). The two surface features in the set are the spring (S1) and the inclined plane (S2). Part of the problem set is shown in matrix form:

	D1	D2
S1	S1D1	S1D2
S2	S2D1	S2D2

This matrix includes four problems. The entire set contains four such subsets of four problems. Clearly, the entire set, having 16 problems, contains two different deep structures and two surface features.

The novice sample consists of 23 Physics 221 students and the expert sample includes five Ph.D. physicists.

Information for Post Hoc Analysis

In an effort to glean information concerning possible differences in categorization of mechanics problems among novices, the final grades in Physics 221, the ACT science scores, and the high school class ranks of the novices were obtained. This study, while concentrating on expert-novice differences, also explores some speculative questions dealing with differences among novices such as "Is there a relationship between higher ACT scores and a categorization pattern that includes more 'expert-like' features than a 'typical novice-like' pattern?" and "Is there a relationship between 'expert-like' features and higher physics course grades?"

Treatment of Data

The study utilized cluster analysis (Euclidean distance) as the tool for the analysis of the data.

Suppose that eight subjects categorize eight mechanics problems. The categorization process yields results as shown in Figure 13A. Cluster analysis is a process that allows for the arrangement of objects, physics problems in this case, into subsets or clusters. The problems within a cluster are more homogeneous than they would be if they were compared to problems that belong to other clusters. Consider Problem 1 and Problem 2 in Figure 13A. Notice that these two problems differ in the numbers of subjects who categorized them as belonging to the categories:

Second Law, Energy Principles, Momentum Principles, and Conservation of Angular Momentum. It is easy to visualize the distance between Problem 1 and Problem 2 if only the first three categories - Second Law, Energy Principles, and Momentum Principles - are considered. Figure 13B clearly shows that Problem 1 can be represented by the coordinates

Problems	1	2	3	4	5	6	7	8
Second Law	2	1	0	4	3	2	2	1
Energy Princ.	4	2	0	4	3	0	2	0
Moment. Princ.	1	5	0	0	1	0	2	0
Angular Motion	0	0	1	0	0	0	0	0
Circular Motion	0	0	2	0	0	0	0	5
Center of Mass	0	0	0	0	0	0	0	0
Linear Kinem.	0	0	0	0	0	0	0	0
Cons. of Ang. Mom.	1	0	5	0	1	0	2	2
Number of Subjects	8	8	8	8	8	8	8	8

Figure 13A. Data matrix

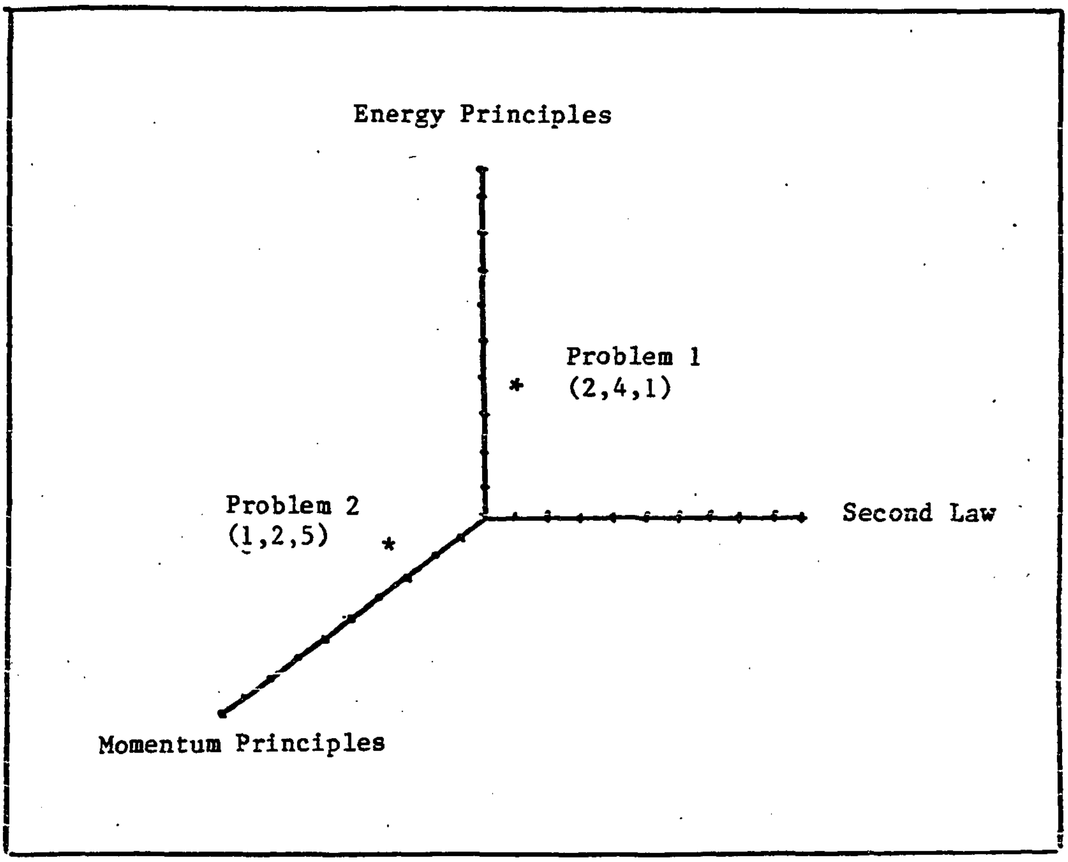


Figure 13B. Representation of two mechanics problems

(2,4,1) in a three-dimensional space bound by the three categories. Similarly, the coordinates (1,2,5) fix Problem 2. The Euclidean distance between the two problems is then calculated: d(1,2) = root of {(2 - 1) squared + (4 - 2) squared + (1 - 5) squared} = 4.58. When problems differ across four or more categories, it is difficult or impossible to visualize the relative distances of the points, each of which represents a problem, in space that exceeds three dimensions. The process may be formalized. According to Hinz (1973):

> Although distances in these higher dimensions are impossible to visualize geometrically, they are easily calculated for any two objects. If the _n_ measurements for object _i_ are given in the n x 1 vector X(i), then the distance from object _i_ to object _j_ , d(ij), is calculated as
>
> d(ij) = root of {(X(i) - X(j)}'{X(i) - X(j)}.
>
> This calculation is very simple and can be performed for all possible pairs of objects. The distances can then be arranged in what is known as an association matrix (p. 113).

The above formula is used to accomplish a transposition of a data matrix, such as shown in Figure 13A, to an association matrix. Figure 14 illustrates this use of the formula in detail for a few examples. When all four categories are considered, the Euclidean distance between Problem 1 and Problem 2 is calculated as shown in Figure 14. Similarly, all possible distances between Problem 1 and the other problems, between Problem 2 and the other problems, etc. are calculated. These distances then are arranged in an association matrix, M 1, as shown in Figure 15. Notice that the distance between Problem 1 and Problem 5, 1.41, is the shortest distance between any two problems (single linkage cluster

$$d_{ij} = \sqrt{(S_i - X_j)^2 \; (X_i - X_j)}$$

$$(X_i - X_j) = \begin{bmatrix} 2 \\ 4 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -4 \\ 1 \end{bmatrix}$$

$$(S_i - X_j'(S_i - X_j) = [\ 1\ \ 2\ \ -4\ \ 1] \begin{bmatrix} 1 \\ 2 \\ -4 \\ 1 \end{bmatrix}$$

$$(X_i - X_j)'(X_i - X_j) = (1)(1) + (2)(2) + (-4)(-4) + (1)(1)$$

$$(X_i - X_j)'(X_i - X_j) = 1 + 4 + 16 + 1$$

$$(X_i - X_j)'(X_i - X_j) = \sqrt{22}$$

$$d_{1,2} = \quad 22 \qquad 4.69$$

Similarly, $d_{1,3} = \sqrt{(2-0)^2 + (4-0^2 + (1-0)^2 + (0-2)^2 \div (1-5)^2} = 6.48$

.

.

.

$d_{1,8} = \sqrt{(2-1)^2 + (4-0)^2 + (1-0)^2 + (0-5)^2 + (1-2)^2} = 6.63$

Then, $d_{2,3} = \ 7.75$

$d_{2,8} = \ 7.62$

etc.

(Adapted from Hinz, 1973)

Figure 14. Calculation of distances between problems

Association Matrix, M1

	1	2	3	4	5	6	7	8
1	0							
2	4.69	0						
3	6.48	7.75	0					
4	4.24	6.16	7.87	0				
5	1.41	4.69	6.32	2.00	0			
6	6.16	6.78	7.35	6.32	5.66	0		
7	2.45	3.74	5.10	4.00	2.00	5.66	0	
8	6.63	7.62	4.47	7.35	6.32	7.07	5.83	0

Association Matrix, M2

	1+5	2	3	4	6	7	8
1+5	0						
2	4.69	0					
3	6.32	7.75	0				
4	2.00	6.16	7.87	0			
6	5.66	6.78	7.35	6.32	0		
7	2.00	3.74	5.10	4.00	5.66	0	
8	6.32	7.62	4.47	7.35	7.07	5.83	0

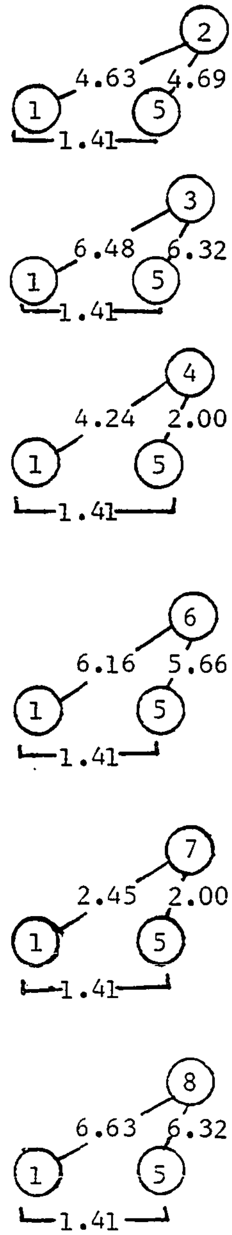


Figure 15. Reduction of an association matrix

M3

	1+5+4	2	3	6	7	8
1+5+4	0					
2	4.69	0				
3	6.32	7.75	0			
6	5.66	6.78	7.35	0		
7	2.00	3.74	5.10	5.66	0	
8	6.32	7.62	4.47	7.07	5.83	0

M4

	1+5+4+7	2	3	6	8
1+5+4+7	0				
2	3.74	0			
3	5.10	7.75	0		
6	5.66	6.78	7.35	0	
8	5.83	7.62	4.47	7.07	0

M5

	1+5+4+7+2	3	6	8
1+5+4+7+2	0			
3	5.10	0		
6	5.66	7.35	0	
8	5.83	4.47	7.07	0

M6

	1+5+4+7+2	3+8	6
1+5+4+7+2	0		
3+8	5.10	0	
6	5.66	7.07	0

M7

	1+5+4+7+2	3+8
1+5+4+7+2	0	5.66
3+8	5.66	0

Figure 16. Further reduction of association matrices

analysis) as obtained by quantitative inspection of M 1. Problems 1 and 5 are then clustered. Figure 16 shows the further reduction of each association matrix and the concomitant clustering. The process ends with the formation of a 2 x 2 matrix.

Association matrices may contain a quantity of information that precludes easy interpretation. The information within matrices may be transformed into dendograms (dendographs). Hinz (1973) holds that the dendogram can be treated as a statistic and, in a sense, summarizes the information in a data matrix such as the one shown in Figure 13A. The objects, physics problems in this case, are represented as equidistant points along an abscissa (see Figure 17). The measure of similarity is plotted along the ordinate; in this case the measure of similarity is the distance between problems belonging to different categories. Vertical lines are drawn from points 1 and 5. A horizontal line at the height of 1.41, the distance between problems 1 and 5, connects the vertical lines. The distance between the 1,5 cluster and Problem 4 is 2.00 as shown in M 2 in Figure 15. Vertical lines are drawn from the 1,5 cluster and Problem 4 and joined by a horizontal line at an ordinate of 2.00. Continuation of this process yields the dendogram shown in Figure 17.

Problems in the dendogram with greatest similarity are connected at lower ordinal values. Clusters that do not overlap may be obtained by choosing a level of dissimilarity (Hinz, 1973). A horizontal line drawn at 2.50 would result in 5 subsets: (1, 5, 4, 7), 2, 3, 8, and 6 while such a line drawn at 5.00 would yield 3 subsets: (1, 5, 4, 7, 2), (3,

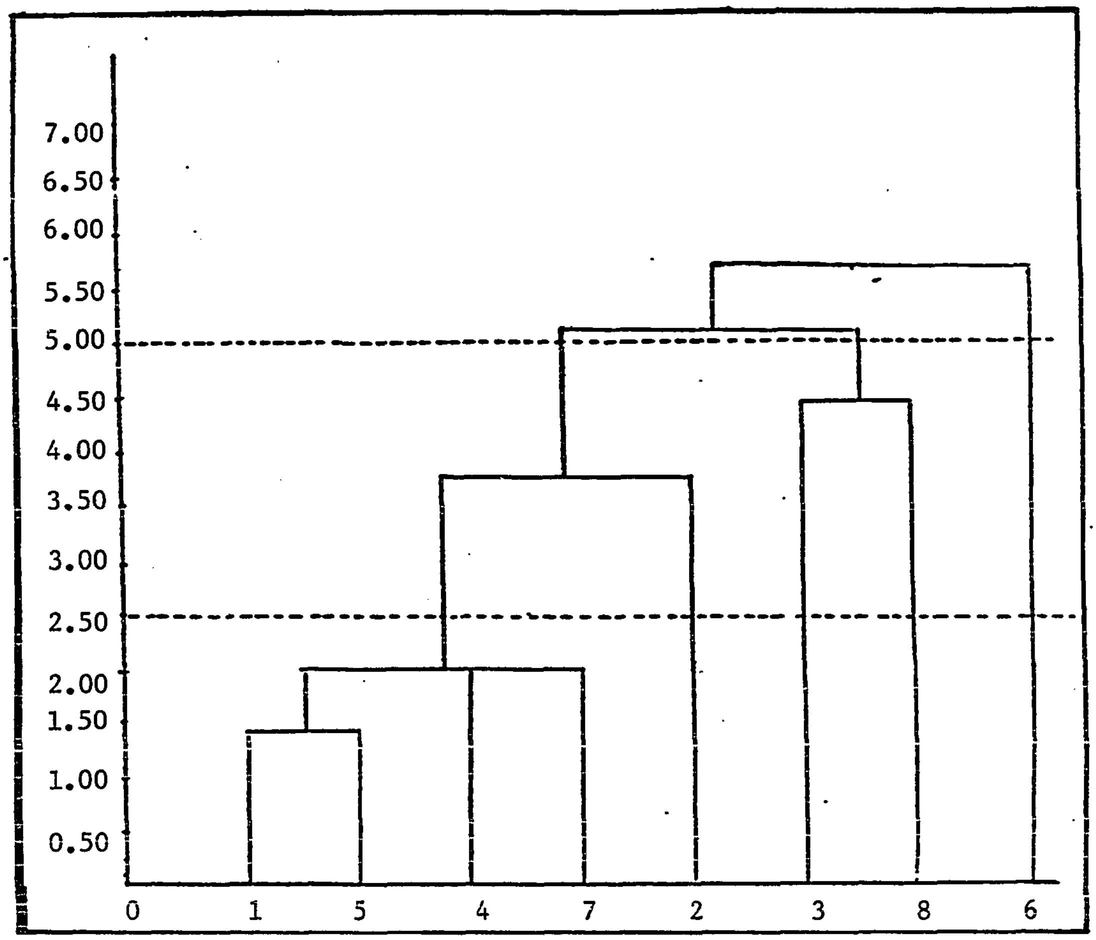


Figure 17. Dendogram derived from association matrices

8), and 6.

The CLUSTER procedure (SAS, 1985) was used. The computer outputs include dendograms in which the minimal distances between clusters (ordinal values), as explained in the expository example above, have been calculated as normalized values, thus making comparisons among different task results possible.

Each problem is described by 17 variables (four expert categories, 12 novice categories, and one No Category). One subject categorizing one problem constitutes one category use. When the rationale card for a subset includes two categories, one half category use is entered under each category. There are 2368 possible category uses (33, 25, 33, and 28 subjects sorting the task sets of 24, 24, 16, and 16 problems, respectively). The input for the cluster-analysis process for each problem is a data line that consists of the number of category uses for each of the 17 variables.

Categories used extensively and labeled by the expert subjects are Newton's Second Law (FMA), Conservation of Energy (CE), Conservation of Linear Momentum (CLM), and Conservation of Angular Momentum (CAM). The four categories associated with these labels are considered to be expert categories as they contain 92 % of the category uses made by the expert subjects.

Twelve other categories are Angular Motion (AMOT), Center of Mass (COM), Frames of Reference (FRM), Springs (SP), Inclined Plane (PL), Pulleys (PUL), Terms (TM), Friction (FR), Atwood Machine (ATW), Kinematics (KIN), Energy (E), and Vectors (VEC). Terms (TM) includes physical arrangements of objects and literal physics terms that are included in the sorted problems. These categories contain only 5 % of the category uses made by the expert subjects. Subsequently, these 12 categories constitute the novice categories in this study.

No Category (NO) is the 17th category which includes 3 % of the category uses made by the expert subjects.

A criterion for cluster labeling allows for meaningful interpretation of the data: The label of a cluster is a function of the categories imposed on the problems in that cluster by <u>at least 75 %</u> of the subjects who have sorted the problem set of which the cluster is a part. The use of a 75 % subject participation in the cluster-labeling process, although arbitrary, allows for inclusion of two or three categories (four categories in case of a tie between the third and fourth category) most frequently used by subjects. Labels used relatively infrequently are thus eliminated.

When combinations of labels associated with a cluster occur, the label used by the greater percentage of subjects who sorted the problem set of which the cluster is a part becomes the label of the cluster.

The clusters resulting from the sorting by the expert subjects may contain more than one expert category. Illustratively, Figure 18 includes an expert category labeled FMA (Newton's Second Law) with an accompanying indication of 81 %. This is a cluster in which 81 % of the subjects encompassed by the cluster-labeling criterion viewed the cluster to consist of Newton's Second Law problems while 19 % per cent of those subjects judged these problems to belong to one or two expert

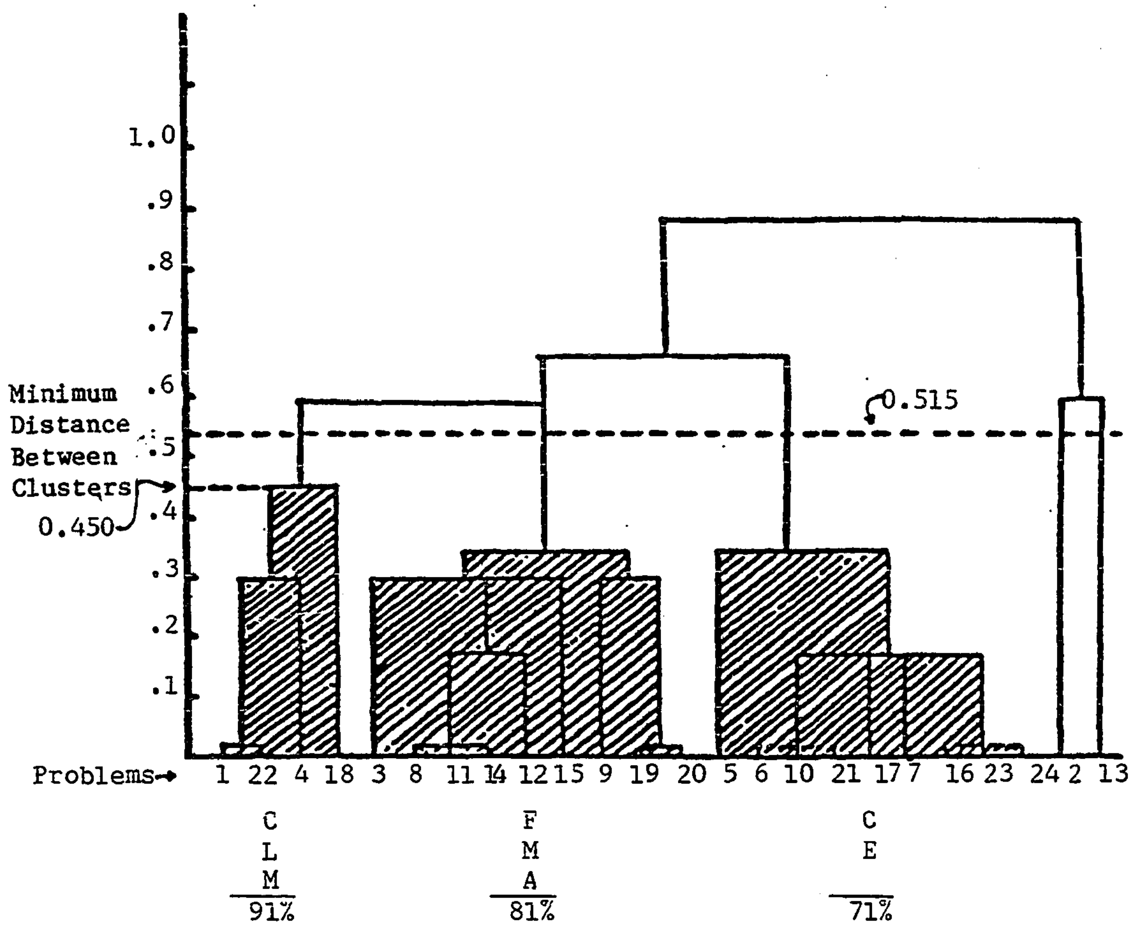


Figure 18. Expert dendogram

Legend: CLM = Conservation of Linear Momentum
FMA = Newton's Second Law
CE = Conservation of Energy

categories which are not included in the Newton's Second Law label.

The clusters resulting from the sorting by the novice and intermediate subjects may also contain more than one category. The sorting process by these subjects resulted in both expert and novice clusters (as opposed to the sorting process by the expert subjects which resulted in only expert clusters). Illustratively, Figure 19 shows a novice dendogram that includes an expert cluster labeled FMA (Newton's Second Law) with an accompanying indication of 73 %. It is a cluster in which 73 % of those subjects encompassed by the cluster-labeling criterion view the problems in the cluster as belonging to the Newton's Second Law category, expert category, while 27 % categorized these problems as belonging to one or more novice categories. On the other hand, also in Figure 19, a novice category labeled SP (Springs) with an accompanying indication of 88 % is a novice cluster in which 88 % of the subjects encompassed by the cluster-labeling criterion have categorized the problems in the cluster as belonging to the Springs cluster, novice category, while 12 % of the subjects categorized these problems as belonging to one or more expert categories. The novice and intermediate dendograms include expert clusters which are shaded with slanting lines and novice clusters which are shaded with horizontal lines.

Various interpretations can result from dendograms. A criterion for inclusion of clusters in the testing of hypotheses is used in this study: A cluster is identified as a hypothesis-testing cluster when it, in its entirety, exists below the ordinal value (minimal distance

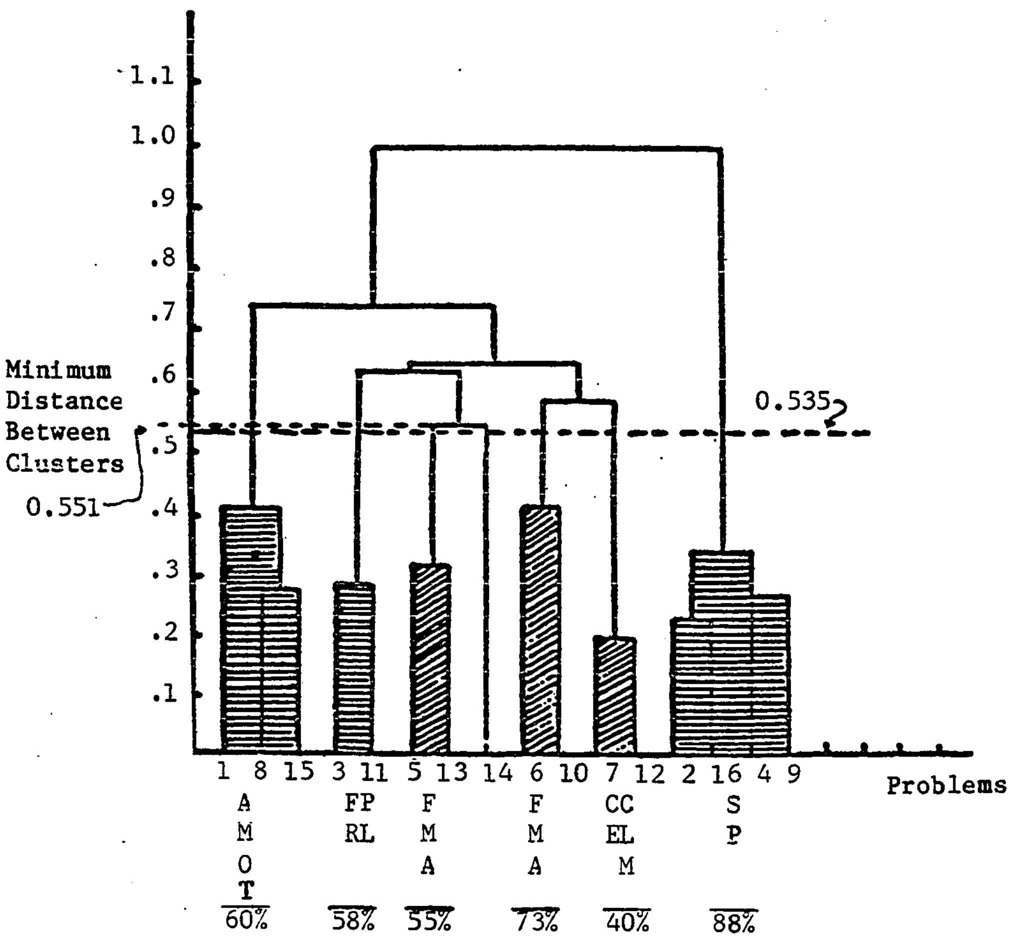


Figure 19. Novice dendogram

Legend: AMOT = Angular Motion
FR,PL = Friction and Inclined Plane
FMA = Newton's Second Law
CE,CLM = Conservation of Energy and Conservation of Linear Momentum
SP = Springs

between clusters) of 0.535 in the dendograms.

The 0.535 value was selected because it allows for the identification of the four expert categories (deep structures) built into the third problem set. The expert subjects who sorted this set (Task 3) categorized the problems in a way that, at the 0.535 value, yields the same four categories with the same labels as those that were built into the set. The built-in categories were validated by two university physicists who solved and categorized each problem in the set. The results attained by the expert subjects assigned to this set, which are the same as those attained by the two validating physicists, are the basis for comparisons among the task results. Clearly, different ordinal values would reveal different sets of clusters. The 0.535 value is applied to all dendograms in the study and serves as a reference line. A dendogram may be considered a statistic but, according to Hinz (1973):

> ...attempts to include probabilistic elements into cluster analysis methods have largely been unsuccessful. Thus confidence intervals, tests of significance, maximum likelihood estimation, etc. are features common to statistical methodology but are generally not available in cluster analysis.
>
> Despite the obvious lack of theoretical basis, cluster analysis techniques have proven to be successful in a wide variety of data analysis situations (p. 121).

In the absence of statistical tests, a dendogram must be interpreted from a well-defined perspective. The testing of the hypotheses in this study is accomplished within the limitations imposed by the two criteria: At least 75 % of the subjects participate in the

At least 75 % of the subjects participate in the cluster-labeling process and only those clusters below the 0.535 reference line are included as hypotheses are tested.

A detailed example of the cluster-labeling process simplifies the next chapter in which the results are discussed.

The novice dendogram in Figure 19, the results of Task 3 accomplished by the novice subjects assigned to Problem Set 3, includes six clusters below the 0.535 reference line. Three of these clusters are expert clusters (slanted shading) and three are novice clusters (horizontal shading). The cluster composed of problems 1, 8, and 15 is labeled AMOT (Angular Motion). This particular cluster is used illustratively in a detailed explanation of the application of both criteria to novice results. With 23 novice subjects categorizing three problems, the total number of category uses is 69. The data lines for these problems are given below, marked with the labels FMA (Newton's Second Law), CE (Conservation of Energy), CLM (Conservation of Linear Momentum), CAM (Conservation of Angular Momentum), AMOT (Angular Motion), COM (Center of Mass), FRM (Frames of Reference), SP (Springs), PL (Pulleys), TM (Terms), FR (Friction), ATW (Atwood Machine), KIN (Kinematics), E (Energy), VEC (Vectors), and NO (No Category). The first, second, third, and fourth data lines, respectively, contain the category uses for Problems 1, 8, 15, and the total category uses for these three problems.

FMA	CE	CLM	CAM	AMOT	COM	FRM	SP	PL	PUL	TM	FR	ATW	KIN	E	VEC	NO
0	4	0	4.5	7	0	0	0	0	0	3.5	0	0	0	0	0	4
.5	1	.5	4.5	13.5	0	1	0	0	2	0	0	0	0	0	0	0
.5	1.5	0	5.5	11.5	0	0	0	1	0	1	0	0	0	0	0	2
1	6.5	.5	14.5	32.0	0	1	0	1	2	4.5	0	0	0	0	0	6

The criterion of cluster labeling is met as 53 responses are considered in the labeling determination (53/69 x 100 % = 77 %). The ratio AMOT (Angular Motion, a novice category) : CE (Conservation of Energy, an expert category) and CAM (Conservation of Angular Momentum, an expert category) = 32.0 : (6.5 +14.5) = 1.5 : 1.0. This cluster ratio determines the label and relative composition of the cluster: AMOT (Angular Motion) is a novice cluster that includes 60 % novice categorical content and 40 % expert categorical content. It is shaded with horizontal lines in the dendogram.

The cluster composed of Problems 5 and 13 is labeled FMA (Newton's Second Law). It is an expert cluster that contains 55 % expert categorical content and 45 % novice categorical content. It is shaded with slanting lines in the dendogram.

Figure 19 contains the labeled clusters, with their respective percentages indicating the dominant categorical content, AMOT (Angular Motion), FR and PL (Friction and Inclined Plane), FMA (Newton's Second Law), FMA (Newton's Second Law), CE and CLM (Conservation of Energy and

Conservation of Linear Momentum), and SP (Springs). There are two Second Law clusters: They share the same dominant expert content but differ in novice content. The first Second Law cluster includes 55 % expert categorical content and 45 % novice categorical content and the other cluster has 73 % expert categorical content and 27 % novice categorical content.

The expert dendogram in Figure 20 also includes the cluster composed of Problems 1, 8, and 15. The experts labeled this set of three problems CAM (Conservation of Angular Momentum). This resulting label emerges from the data on which the cluster is based. As this is an expert dendogram, the 86 % indication means that the dominant expert categorical content constitutes 86 % and the expert content not used in the determination of the label accounts for 14 % of the total number of category uses allowed by the cluster-labeling criterion.

Description of Design and Post Hoc Treatment of Data

A dependent variable, DEGREE, was designed for investigating possible relationships between "expert-like" behavior by the novices and ACT science scores, the final grades in Physics 221, and the percentile ranks in the high school class.

The expert subjects demonstrated a good match between choice of category and solution in terms of that category; 87.5 % of them showed a perfect match.

Each subject in this study solved one of the problems in the particular problem set which he/she sorted. Each solved problem was

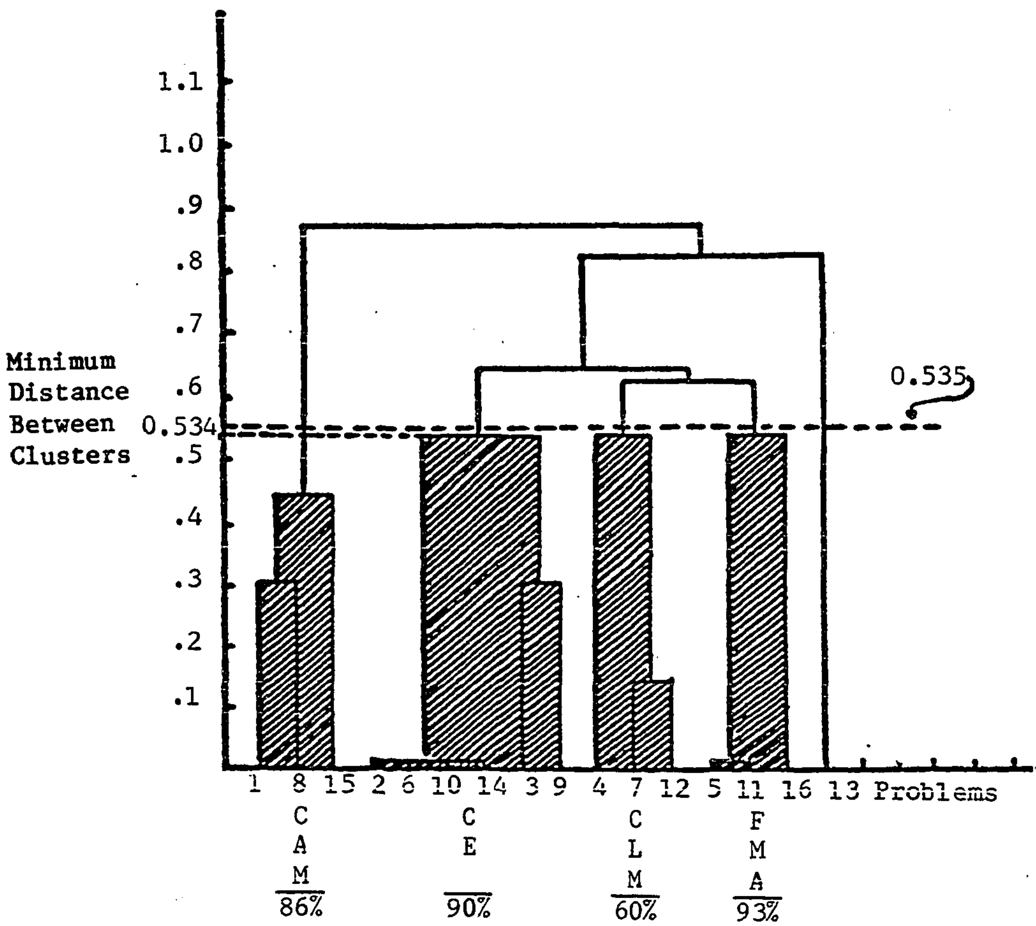


Figure 20. Expert dendogram

Legend: CAM = Conservation of Angular Momentum
CE = Conservation of Energy
CLM = Conservation of Linear Momentum
FMA = Newton's Second Law

checked against the category imposed on the problem.

The value of DEGREE is obtained according to the model:

A. Does the solution fit the
imposed category?

yes	=	1.0
partly	=	0.5
no	=	0

B. Is the imposed category
an "expert" category
and does this category
lead to a correct solution?

yes (and correct solution)	=	1.0
yes (but incorrect solution)	=	0.5
no	=	0

The score on the DEGREE variable = DEGREE = Score A + Score B.

The results of the imposed category/solution matches by the expert subjects in this study were noted and expressed numerically.

The independent variables and their levels are:

ACT science score

greater than or equal to 33 3
less than or equal to 32 to
greater than or equal to 28 2
less than or equal to 27 1

Final grade in Physics 221
(scale: A = 11, B+ = 10 ...

D- = 1, F = 0)

7 - 11 3

4 - 6 2

0 - 3 1

High school class rank (expressed as a percentile)

Upper third of novice sample 3

Middle third of novice sample ...2

Lower third of novice sample 1

The research question, addressed by the post hoc analysis, is: "Are there differences in average DEGREE scores attributable to the ACT science score, the final grade in Physics 221, and the high school class rank?" The accompanying null hypotheses were tested:

HO(1). There are no significant differences in average DEGREE scores among the students when categorized on the basis of the ACT science score, the final grade in Physics 221, and the high school class rank.

HO(2). There is no significant interaction in average DEGREE scores among the students when categorized on the basis of the ACT science score, the final grade in Physics 221, and the high school class rank.

The corresponding alternative hypotheses are:

HA(1). There are significant differences in average

DEGREE scores among the students when categorized on the basis of the ACT science score, the final grade in Physics 221, and the high school class rank.

HA(2). There is significant interaction in average DEGREE scores among the students when categorized on the basis of the ACT science score, the final grade in Physics 221, and the high school class rank.

A Pearson correlation between the DEGREE scores and the final grades in Physics 221 was calculated.

CHAPTER IV. RESULTS

The rationale cards indicate that 95 % of the subjects viewed the sorting tasks from a problem-solving perspective, i.e., the problems were categorized on the basis of perceived solutions to the problems as if they were to be solved. Comments such as "The solution is made easier by ..." and "All of these can be solved by F = ma" are illustrative.

Expert clusters in the dendograms are associated with deep structures (experts are expected to categorize on basis of deep structures) and novice clusters are associated with surface features (novices are expected to categorize on basis of surface features).

Results - Hypothesis 1

Experts will categorize physics (mechanics) problems on basis of deep structures and novices will categorize these problems on basis of surface features.

Chi et al. used comparisons among problems with the greatest measure of agreement and frequency distributions of the number of times the problems in a set were placed in a given category. They were able to accept the hypothesis.

Chi et al. (1981) state that:

> One way to interpret the cluster analysis is to examine only those problems that were grouped together with the highest degree of agreement among subjects (p. 124).

The initial analysis in the Chi et al. study focused on pairs of problems. One pair contains rotating things and was grouped together by all eight novices who sorted the set. Figure 21, containing the novice results of Task 1, shows the AMOT (Angular Motion) cluster to contain Problems 2 and 13 (both deal with rotating things). Problem 2 is identical to one of the two Chi et al. problems and Problem 3 is similar to the other in that both deal with wheels on shafts with a given angular velocity. The sorting outcomes in both studies are similar. Such a comparison between studies can be made but is complicated by the use of problems that, occasionally, are not identical.

Eight novice subjects participated in the Chi et al. study while this study includes data from 94 novice subjects. This study uses the hypothesis-testing clusters in each entire dendogram. The use of entire dendograms allows for greater inclusion of data in a clear, demonstrative manner. Individual problems are considered in the analysis of the data resulting from the third problem set (Task 3) and the fourth problem set (Task 4) as these problem sets have identifiable a priori structures.

The expert dendogram in Figure 22, showing results of Task 1, contains no novice clusters and three expert clusters and supports the hypothesis: Experts categorize the problems on the basis of deep structures.

The novice dendogram in Figure 21 includes six novice clusters and

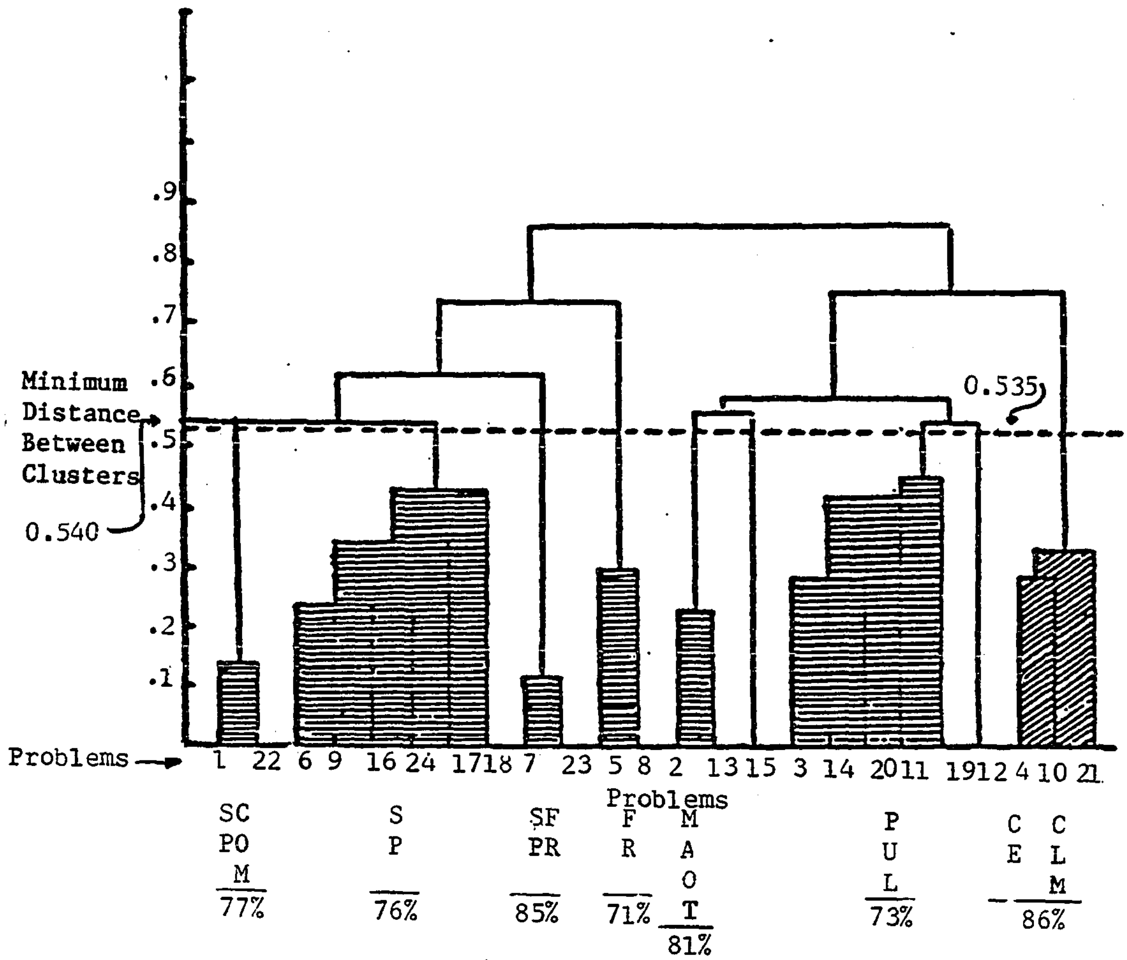


Figure 21. Novice dendogram - Results of Task 1

Legend: SP. COM = Springs and center of mass
SP = Springs
SP,FR = Friction
AMOT = Angular motion
PUL = Pulleys
CE,CLM = Conservation of Energy and Conservation of Linear Momentum

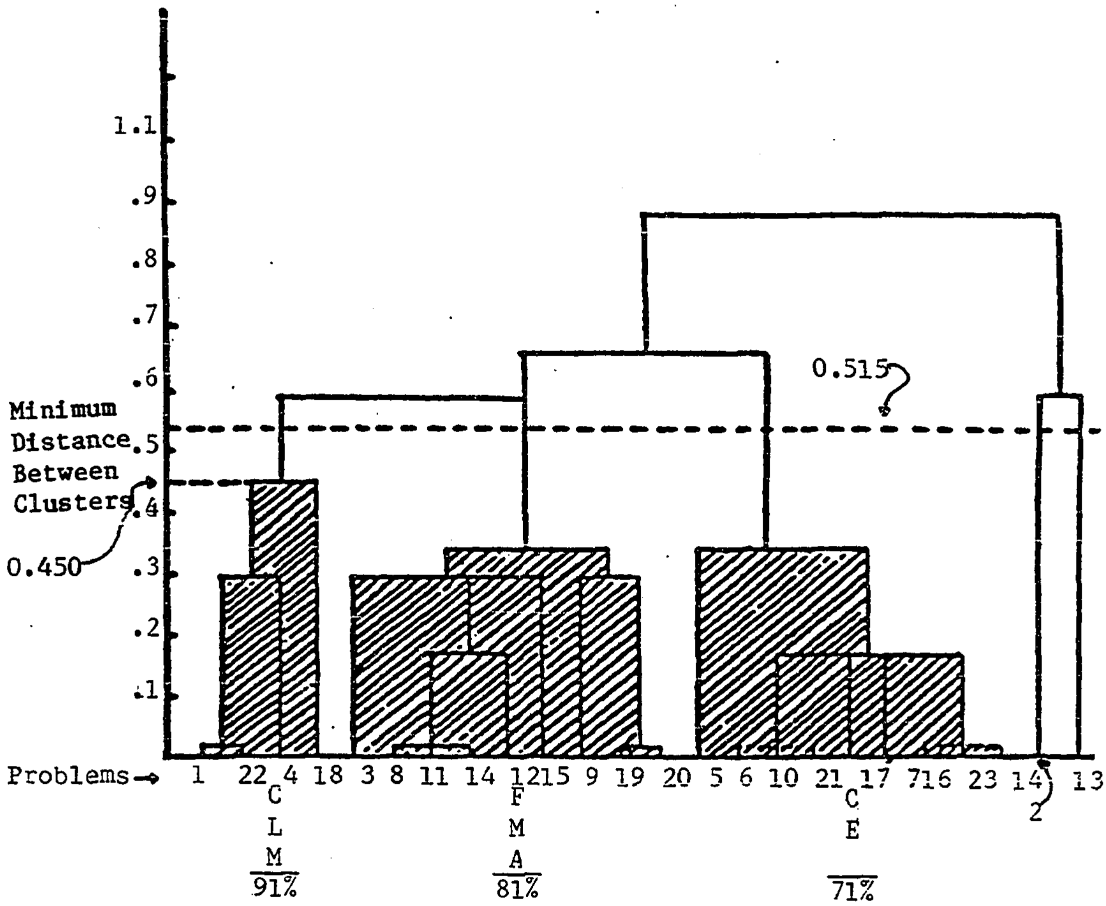


Figure 22. Expert dendogram - Results of Task 1

Legend: CLM = Conservation of Linear Momentum
FMA = Newton's Second Law
CE = Conservation of Energy

one expert cluster and supports the hypothesis: Novices categorize the problems on the basis of surface features.

The analysis of pairs of problems and the use of frequency distributions of the number of times the problems were placed in a given category by Chi et al. and the cluster analysis with the resulting dendograms in this study both allow for the acceptance of Hypothesis 1: The dendograms constitute a valid method of analysis and the Chi et al. results are confirmed in the replicative testing of Hypothesis 1.

Results - Hypothesis 2

Experts will categorize a different set of physics (mechanics) problems on basis of deep structures and novices will categorize this set on basis of surface features.

The expert dendogram in Figure 23, showing results of Task 2, contains no novice clusters and three expert clusters and supports the hypothesis: Experts categorize the problems on the basis of deep structures.

The novice dendogram in Figure 24, showing results of Task 2, contains three novice clusters and two expert clusters. With ten problems in the two expert clusters (by comparison, the results of Task 1 shown in Figure 21 include three problems in its single expert cluster), the dendogram does not support the hypothesis.

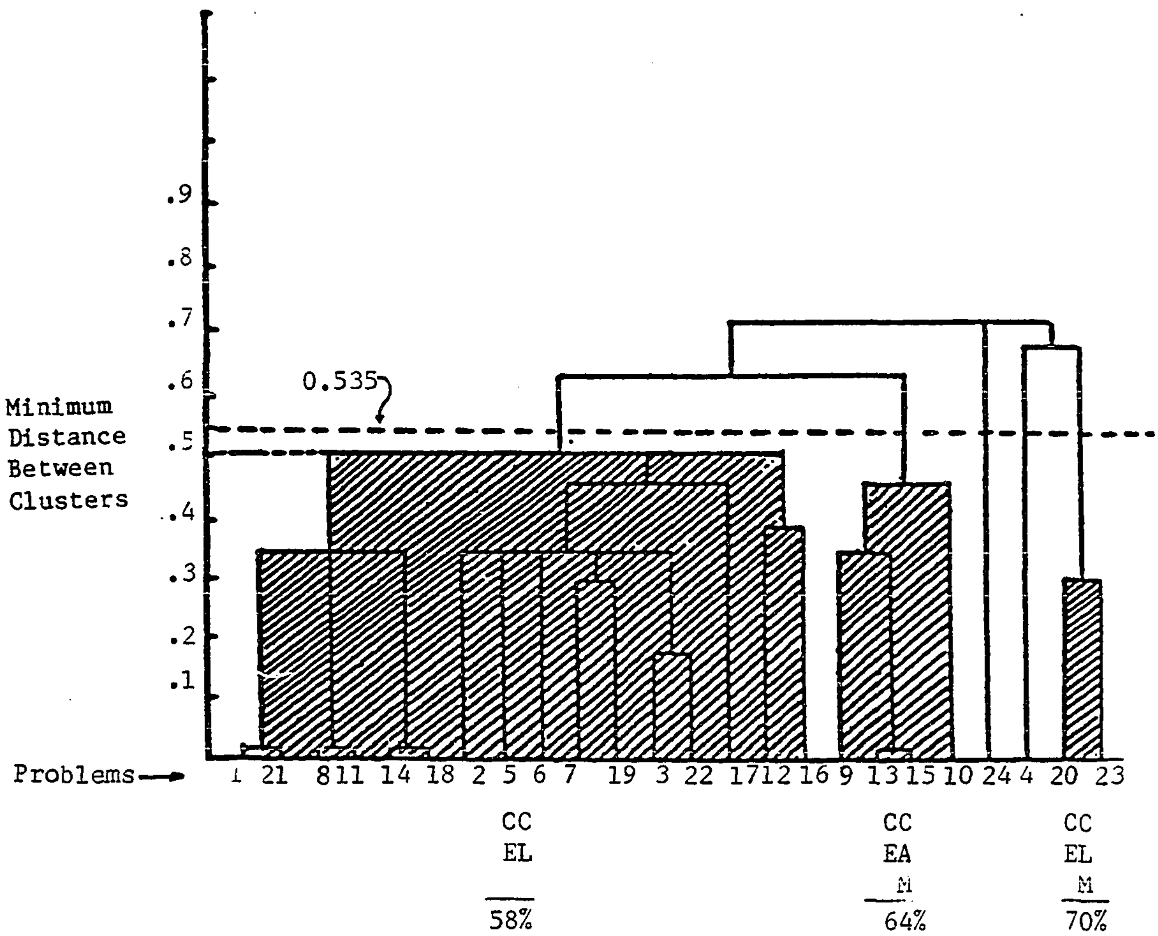


Figure 23. Expert dendogram - Results of Task 2

Legend: CE,CLM = Conservation of Energy and Conservation of Linear Momentum
CE,CAM = Conservation of Energy and Conservation of Angular Momentum
CE,CLM = Conservation of Energy and Conservation of Linear Momentum

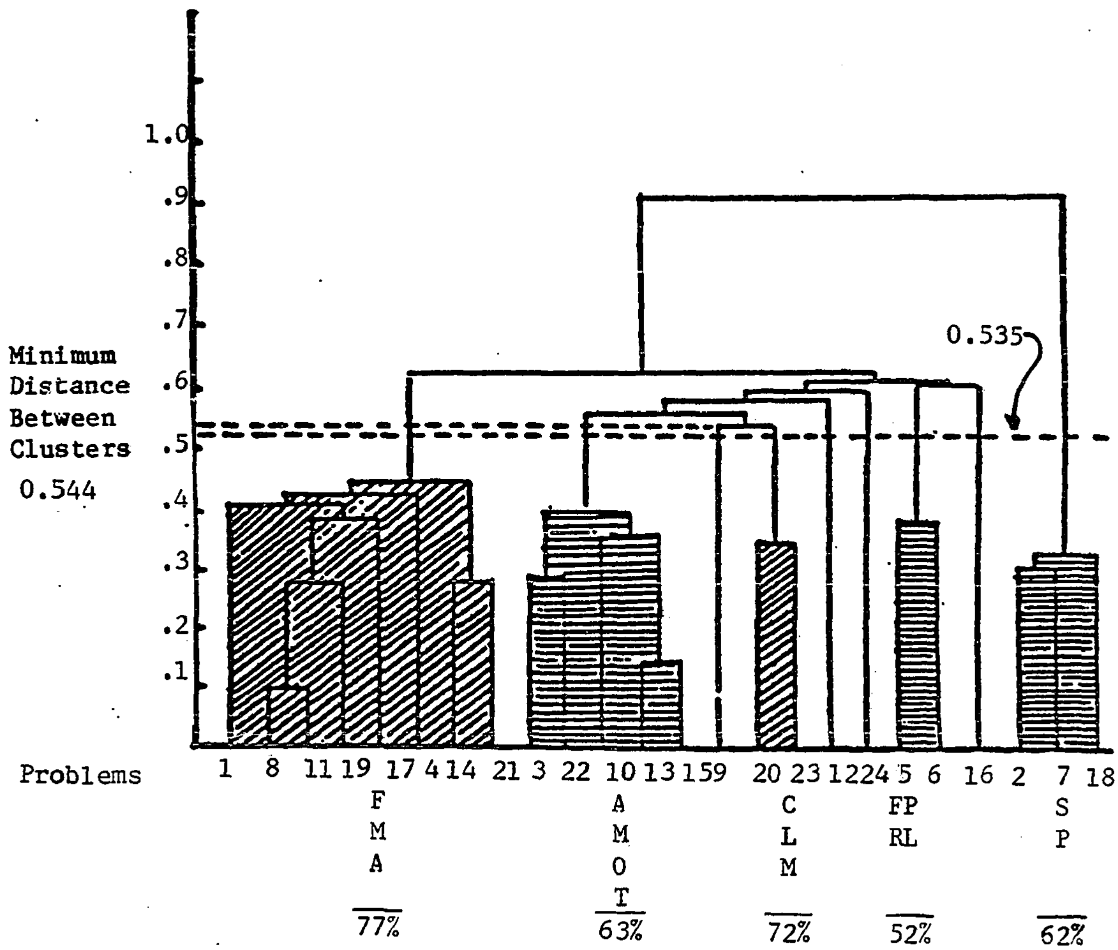


Figure 24. Novice dendogram - Results of Task 2

Legend: FMA = Newton's Second Law
AMOT = Angular Motion
CLM = Conservation of Linear Momentum
FR,PL = Friction and Inclined Plane
SP = Springs

Results - Hypothesis 3

Experts will categorize physics problems (mechanics) problems according to deep structures regardless of surface features and novices will categorize these problems according to surface features regardless of deep structures. Intermediates will reveal a categorizing pattern that is characterized by a mixture of deep structures and surface features.

The expert dendogram in Figure 25 shows results of Task 3. This is the set with four deep structures and four surface features. Each problem contains one deep structure that is counterbalanced with one surface feature.

The expert subjects labeled the problems with a clustering pattern that reflects the built-in deep structures with some exceptions. The a priori categories (as based on the construction and validation of the set) as compared to the resulting clusters labeled by the subjects are: Conservation of Angular Momentum (1, 8, 9, 15 vs. 1, 8, 15), Conservation of Energy (2, 6, 10, 14 vs. 2, 6, 10, 14, 3, 9), Conservation of Linear Momentum (4, 7, 12, 13 vs. 4, 7, 12), and Newton's Second Law (3, 5, 11, 16 vs. 5, 11, 16). Problem 9 is a two-step problem, CE (Conservation of Energy) and CAM (Conservation of Angular Momentum), and its place in the CE (Conservation of Energy) cluster may be considered to satisfy the a priori-based expectation. Thus, with the exception of Problems 3 and 13, the subjects labeled the problems according to the a priori-based scheme.

The results of the Task 3 support the hypothesis: Experts

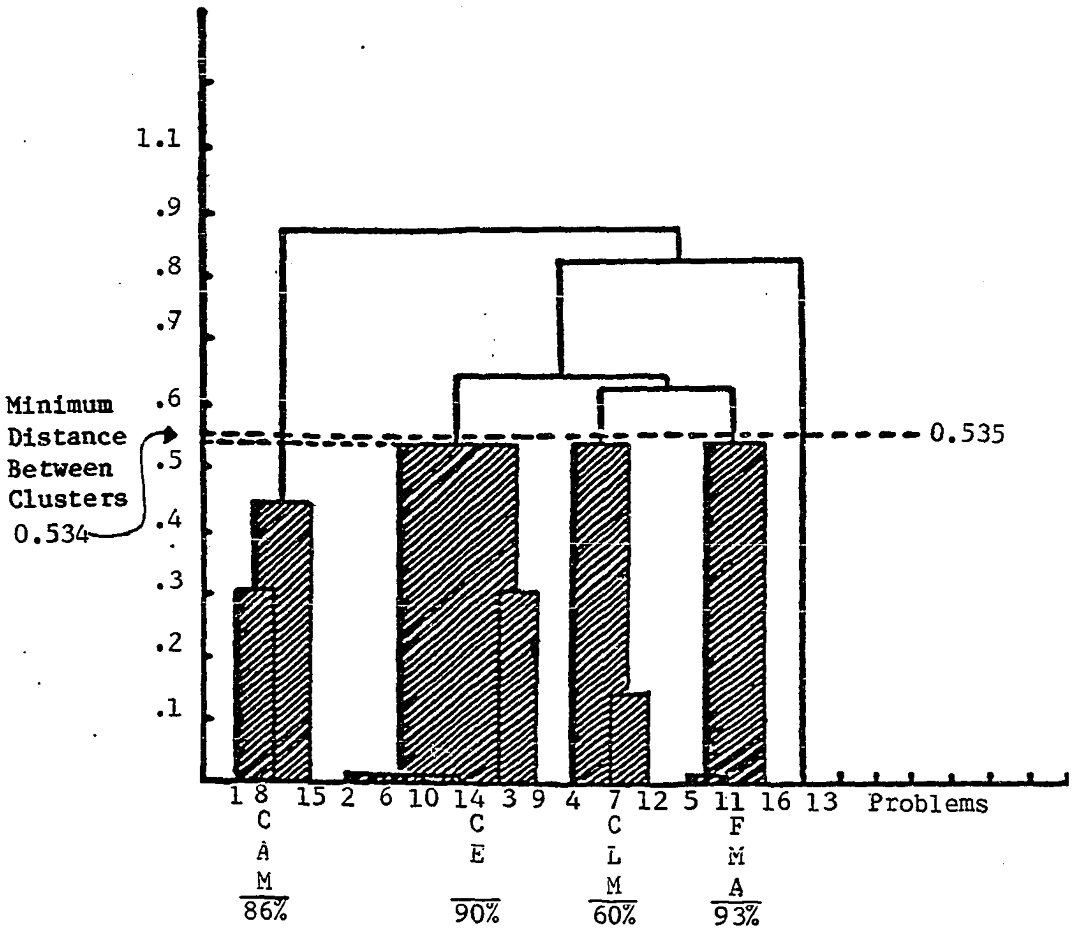


Figure 25. Expert dendogram - Results of Task 3

Legend: CAM = Conservation of Angular Momentum
CE = Conservation of Energy
CLM = Conservation of Linear Momentum
FMA = Newton's Second Law

categorize physics problems according to deep structures regardless of surface features.

The novice dendogram in Figure 26, showing results of Task 3, with three novice clusters and three expert clusters, includes more divergence from the a priori clusters (as based on the construction of the set) than does the expert dendogram.

A few representative cases are discussed. The clusters labeled by the subjects show only the SP (Springs) cluster having a perfect match (2, 4, 9, 16 compared with 2, 16, 4, 9). Two expert clusters, the first FMA (Newton's Second Law) cluster and the CE, CLM (Conservation of Energy and Conservation of Linear Momentum) cluster include Problems 5, 7, and 12. These problems are, respectively, in the PUL (Pulleys), TM (Terms), and PL (Inclined Plane) categories (novice categories) in the a priori structure. The FR, PL (Friction and Inclined Plane) cluster, a novice cluster, in the dendogram contains only Problem 11 of the priori PL (Inclined Plane) category which includes Problems 6, 11, 12, and 15. Problems 6 and 12 are, respectively, in two expert clusters, the second FMA (Newton's Second Law) and the CE, CLM (Conservation of Energy and Conservation of Linear Momentum) clusters. Problem 15 is in the novice AMOT (Angular Motion) cluster. The novice subjects chose some expert labels for problems that contain specific surface features and assigned some novice labels to problems that contain surface features other than those indicated by the assigned labels.

The existence of three expert categories and the divergence from the a priori structure in the novice dendogram do not support the

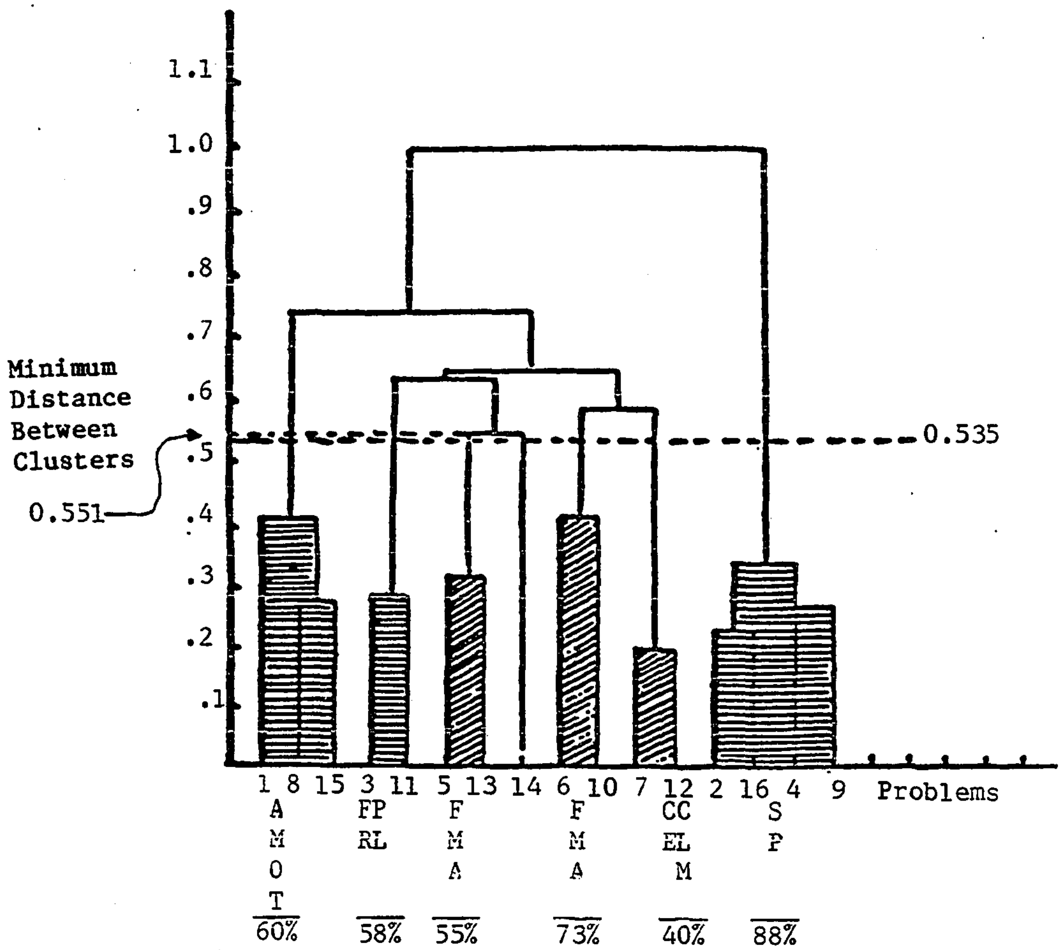


Figure 26. Novice dendogram - Results of Task 3

Legend: AMOT = Angular Motion
FR,PL = Friction and Inclined Plane
FMA = Newton's Second Law
CE,CLM = Conservation of Energy and Conservation of Linear Momentum
SP = Springs

hypothesis.

The intermediate dendogram in Figure 27, showing results of Task 3, with one novice cluster and three expert clusters, resembles the expert dendogram in Figure 25 in that both dendograms consist largely of expert clusters (one novice cluster in the intermediate dendogram). Problems 1, 8, and 15, CAM (Conservation of Angular Momentum), in the intermediate dendogram were clustered and labeled as was done by the experts. The intermediates clustered and labeled Problems 2 and 9, in the CE, CAM (Conservation of Energy and Conservation of Angular Momentum) cluster, approximately as was done by the experts who placed Problems 2 and 9 in the CE (Conservation of Energy) cluster. Problems 7 and 12, CE and CLM (Conservation of Energy and Conservation of Linear Momentum), in the intermediate dendogram were categorized and labeled approximately as was done by the experts who placed Problems 7 and 12 in the CLM (Conservation of Linear Momentum) cluster.

Summarizing, the intermediates categorized and labeled seven of the nine clustered problems in approximately the same manner as did the experts.

The intermediate dendogram includes a novice cluster. Such clusters do not appear in expert dendograms.

Analysis of the intermediate dendogram supports the hypothesis: Intermediates will reveal a categorizing pattern that is characterized by a mixture of deep structures and surface features.

The first three hypotheses involving the experts were supported but only the first hypothesis involving the novices was supported. The

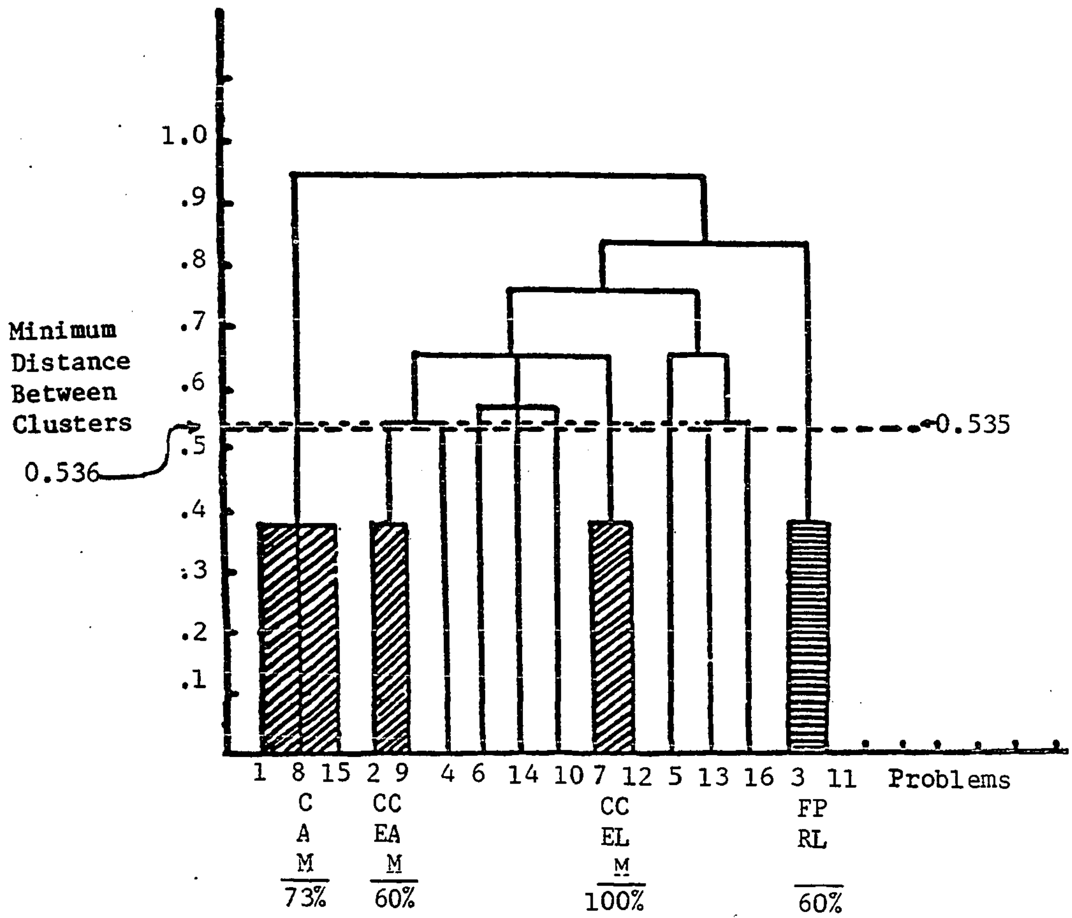


Figure 27. Intermediate dendogram - Results of Task 3

Legend: CAM = Conservation of Angular Momentum
CE,CAM = Conservation of Energy and Conservation of Angular Momentum
CE,CLM = Conservation of Energy and Conservation of Linear Momentum
FR,PL = Friction and Inclined Plane

results indicate that the comparison of the intermediates with the experts is more clearly delineated than it is with the novices.

Results - Hypothesis 4E

Experts will categorize a set of physics (mechanics) problems according to deep structures regardless of surface features with the number of established categories approximately equal to the number of deep structures contained within the set.

The expert dendogram in Figure 28, showing results of Task 4, contains three expert clusters. The a priori categories in the fourth set of problems compared with the resulting clusters that were labeled by the experts are Newton's Second Law (1, 3, 4, 8, 9, 10, 13, 14) vs. (1, 10, 9, 4, 3, 14, 8) and Conservation of Energy (2, 5, 6, 7, 11, 12, 15, 16) vs. (2, 12, 15, 5, 7, 13). The subjects grouped Problems 6 and 11 in a second CE (Conservation of Energy) cluster separate from the CE (Conservation of Energy) cluster consisting of seven problems. Problem 13, expected in the FMA (Newton's Second Law) cluster, was categorized with the first CE (Conservation of Energy) cluster and Problem 16 was not clustered. The subjects categorized the problems into the clearly-delineated FMA (Newton's Second Law) cluster and two CE (Conservation of Energy) clusters, which in spite of their anomalous separation, constitute problems that are expected to be CE (Conservation of Energy) problems. With the exception of Problems 13 and 16, the labeled clusters confirm the expectations of the two a priori categories, FMA (Newton's Second Law) and CE (Conservation of Energy).

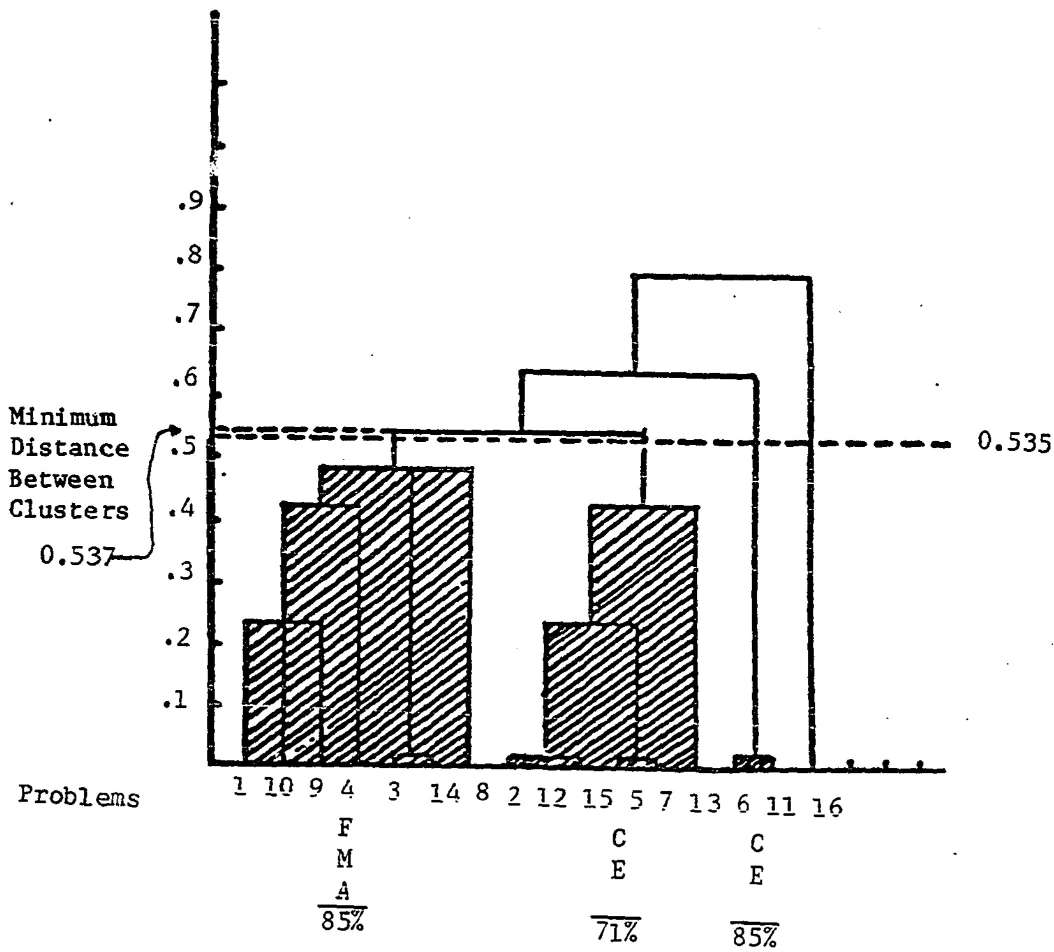


Figure 28. Expert dendogram - Results of Task 4

Legend: FMA = Newton's Second Law
CE = Conservation of Energy

The analysis of the results of Task 4 shows that the a priori structure of two categories in the fourth problem set is satisfied as the dendogram contains the expected clusters. Hypothesis 4 is supported: Experts categorize a set of physics problems according to deep structures regardless of surface features with the number of established categories being approximately equal to the number of deep structures within the set.

Clearly, the expert results of Tasks 3 and 4, with the numbers of clusters (formed by the subjects) being in the ratio of two to one and the number of commensurate categories (a priori) being in the same ratio, show that Hypotheses 3 and 4 serve as checks upon one another. The support for both hypotheses is subsequently strengthened.

Results - Hypothesis 4N

Novices will categorize a set of physics (mechanics) problems according to surface features regardless of deep structures with the number of established categories approximately equal to the number of surface features contained within the set.

The novice dendogram in Figure 29, showing results of Task 4, contains one expert cluster and one novice cluster. The a priori surface features in the fourth problem set are Springs (2, 3, 7, 9, 10, 12, 14, 16) and Inclined Plane (1, 4, 5, 6, 8, 11, 13, 15). Inspection of the dendogram shows that the SP (Springs) cluster is a perfect match with the a priori SP category. The FMA (Newton's Second Law) cluster, while being a perfect match problem-wise with the a priori PL (Inclined

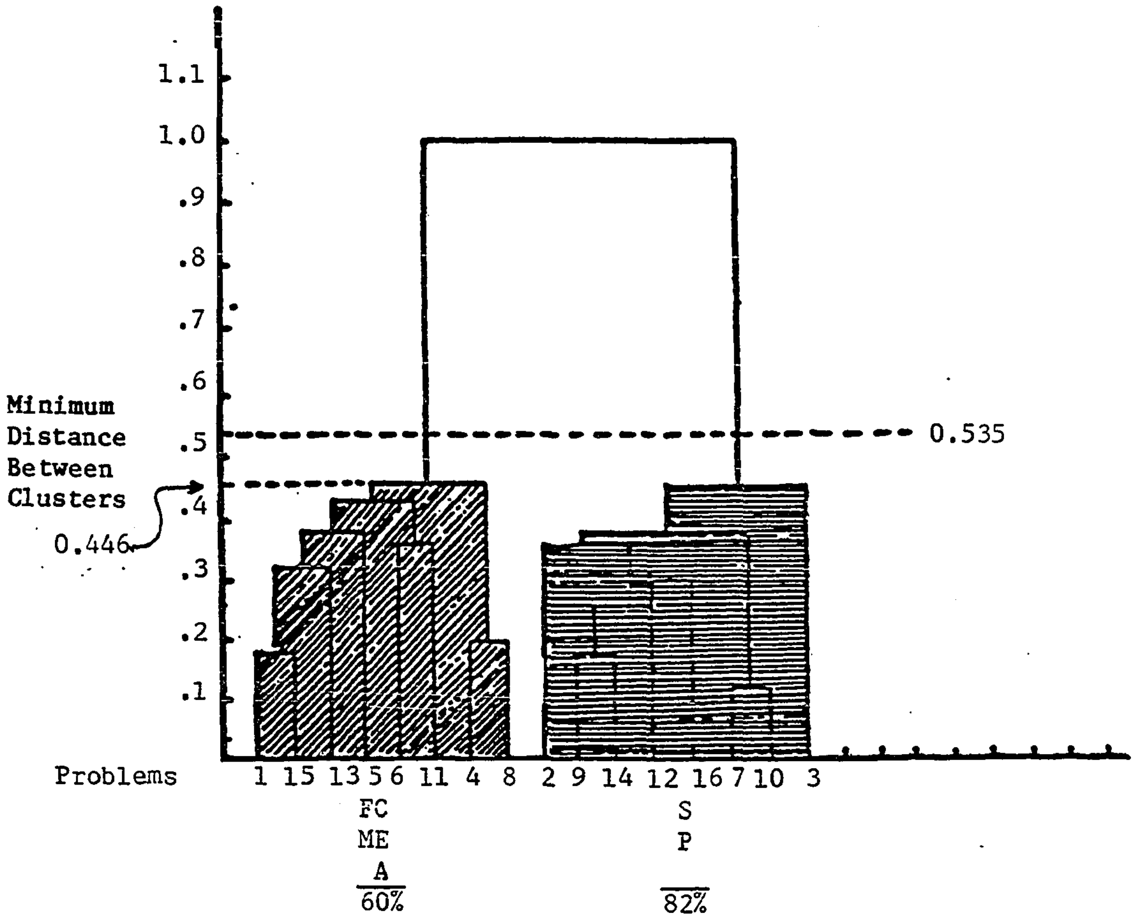


Figure 29. Novice dendogram - Results of Task 4

Legend: FMA, CE = Newton's Second Law and Conservation of Energy
SP = Springs

Plane) category, differs in the label.

The novice results of Task 4 do not support the hypothesis.

Summarizing:

Hypotheses 1, 2, 3, and 4E
(involving the experts)................ accepted

Hypothesis 3
(involving the intermediates).......... accepted

Hypothesis 1
(involving novices)....................accepted

Hypotheses 2, 3, and 4N
(involving novices)....................rejected.

Results - Post Hoc Analysis

The ANOVA procedure (SAS, 1985), with alpha = .05, was used in the testing of the two null hypotheses:

HO(1). There are no significant differences in average DEGREE scores among the novice subjects when categorized on the basis of the ACT science score, the final grade in Physics 221, and the high school class rank.

HO(2). There is no significant interaction in average DEGREE scores among the novice subjects when categorized on the basis of the ACT science score, the final grade in Physics 221, and the high school class rank.

Figure 30A shows the values for DEGREE (the dependent variable), the numbers of subjects in each group, and the levels of the ACT science scores, the final grades in Physics 221, and the high school class ranks (the independent variables). None of the F-values for the main and interaction effects are significant (see Figure 30B), resulting in the statistical conclusion of failure to reject Hypotheses HO(1) and HO(2) with $p < 0.05$. The research is unable to show that there are differences in average DEGREE scores attributable to the ACT science score, the final grade in Physics 221, and the high school class rank.

It is of some interest, however, that the Pearson Correlational Coefficient between the average DEGREE score and the final grade in Physics 221 is 0.21913 at a significance level of 0.0338 which is less than 0.05, the level at which the analysis of variance was run.

DEPENDENT VARIABLE		INDEPENDENT VARIABLES			
DEGREE	N	LEVELS	ACT	GRADE	RANK
2.0	11	3	20	55	69
1.5	25	2	60	30	18
1.0	9	1	14	9	7
0.5	17				
0	32				
	--		--	--	--
	94		94	94	94

Figure 30A. Table: Numbers of subjects and levels of the dependent and independent variables for the analysis of variance

SOURCES OF VARIATION	df	SUM OF SQUARES	F-VALUE	PR> F
EXPLAINED	19	12.8395	1.33	0.1912
RESIDUAL	74	37.5861		
TOTAL	93	50.4255		
ACT	2	2.2136	2.18	0.1203
GRADE	2	3.0233	2.98	0.0571
ACT X GRADE	4	0.6494	0.32	0.8640
RANK	2	0.0134	0.01	0.9869
GRADE X RANK	4	1.9381	0.95	0.4380
ACT X GRADE X RANK	2	0.9638	0.95	0.3919

Figure 30B. Table: Analysis of variance of DEGREE scores by ACT science scores, final grades in physics 221, and high school class rank

CHAPTER V. DISCUSSION

Categorization Patterns of Novices and Experts

Hypothesis 2 was designed to determine whether expert-novice differences in categorization are independent of the first problem set.

The novices sorting the first problem set grouped only three of the 22 clustered problems into the expert CE, CLM (Conservation of Energy and Conservation of Linear Momentum) cluster. Hypothesis 1 (novices) was accepted.

The novices sorting the second problem set grouped ten of the 20 clustered problems into the expert FMA (Newton's Second Law) and CLM (Conservation of Linear Momentum) clusters. Hypothesis 2 (novices) was rejected.

Considering these findings, the novice categorization results are not shown to be independent of the first problem set.

Collectively, the novice dendograms related to Tasks 2, 3, and 4 contain expert clusters with FMA (Newton's Second Law) and CE (Conservation of Energy) categorical content. The 20 problems in these clusters include five PUL (Pulleys), nine PL (Inclined Plane), and six TM (Terms) surface features: The novices imposed expert categories, overriding the surface features, on these problems. The four expert dendograms contain only expert clusters.

Of the hypotheses involving novices, only Hypothesis 1 was accepted. All hypotheses involving experts were accepted. The novice categorization cannot be shown to be independent of the first set. The expert categorization is independent of the first set.

It might be argued that the problems in the first set might be more definitive in surface features than problems in other sets. This possibility is not supported as a few problems common to various sets are traced. The problems in this study are numbered in such a manner that the part of the number before the decimal point refers to the problem set and the part of the number after the decimal point indicates one of the problems in that set, e.g., 1.5 indicates Set 1, Problem 5 and 3.6 indicates Set 3, Problem 6. Problems 1.5, 2.5, and 4.5 are identical. They are in the FR (Friction), FR, PL (Friction and Inclined Plane), and FMA, CE (Newton's Second Law and Conservation of Energy) clusters in the respective dendograms. Problems 2.6, 3.6, and 4.6 are identical and are in the FR, PL (Friction and Inclined Plane), FMA (Newton's Second Law), and the FMA, CE (Newton's Second Law) clusters in the respective dendograms. The same problems are found in clusters that are not the same.

Relationships between Schemata and DEGREE

Chi et al. (1981) hold that the selected categories constitute the schemata that determine the quality of the representation process. Hinsley, Hayes, and Simon (1978) claim that such schemata exist as they show that college students can classify algebra problems in types that are functions of underlying principles. Silver (1981) asked seventh-grade students to sort 16 word problems and to solve 12 of these problems. Analysis of the data showed that the good problem solvers categorized problems primarily by the processes that they intended to use in the subsequent solutions and that the poor problem solvers tended to use the content of the problem statements. It appears that categories are fundamental to problem representations. Representations determine the nature of problem solutions (Newell and Simon, 1972; deKleer, 1977; Novak, 1977; Simon and Simon, 1978; Larkin, 1980, Mayer, 1983).

Two problems used in this study are now considered:

1.5 (2.5) (4.5) A block of mass M starts up an incline of angle theta with respect to the horizontal, with an intial velocity v. How far will it slide up the plane if the coefficient of friction is mu? (Halliday and Resnick, 1974, p. 133).

2.6 (3.6) (4.6) A child of mass M descends a slide of height h and reaches the bottom with a speed of v. Calculate the amount of heat generated.

A group of seventeen novices solved Problem 1.5 (same as 2.5 and 4.5) and a group of sixteen novices solved Problem 2.6 (same as 3.6 and 4.6) with both groups randomly assigned to the problems.

Recall that the DEGREE variable measures the degree to which the solution to a problem fits the imposed category and the degree to which the imposed category is an expert category leading to a correct solution. The scores of the novices were distributed among the possible values of DEGREE (0, 0.5, 1.0, 1.5, and 2.0) with approximately equal frequencies for both groups. Each group of novices categorized and solved a problem that differs in text from that solved by the other group. The numerical differences in the attained DEGREE scores in each group show approximately the same pattern. The experts who sorted and solved the same two problems have, without exception, a value of 2.0 (maximum value) for DEGREE.

A problem solver describes the environment, in this case the statement of a physics problem, and attempts to solve the problem by mental operations on this description, i.e., the reresentation.

Representations are viewed as organized knowledge structures in short-term memory. Knowledge in long-term memory is used in the formation of a problem representation. This knowledge is accessed when a problem solver categorizes a problem. Part of the nature of the schemata in long-term memory may thus be inferred from categorization patterns.

The DEGREE variable, being an operational measure of "expert-like" behavior, describes the type of categorization and the match between categorization and the subsequent solution (see page 83 for method of calculation). Differences in DEGREE values among the novices thus indicate differences in the schemata in their long-term cognitive structure.

It is inferred that differences in DEGREE values across problems (different in text but alike in surface features and deep structures) having approximately the same distributions among the possible values of the variable, indicate differences in the schemata of the subjects.

The rejection of the three hypotheses involving novices is more likely due to differences among the schemata of the novice subjects rather than being caused by differences in the problems in the first set and those in the other three sets. In view of the attained DEGREE scores, the differences among novice schemata are of a lesser degree than the marked differences between novice and expert schemata.

Novice Differences

Eleven novices (12 %) attained a score of 2.0 on the DEGREE variable, i.e., they imposed an expert category on the problem solved by them and subsequently solved the problem correctly within the imposed category: They functioned like the experts in this study. Seventeen novices (18 %), not including the 11 aforementioned subjects, attained a score of 1.0 on the A part of the DEGREE variable, i.e., the solution fits the imposed category: They functioned like the subjects in the Silver (1981) study.

Differences in average DEGREE scores exist but this research, using the analysis of variance, is unable to show that these differences are related to the ACT science score, the final grade in Physics 221, and the high school class rank. However, the Pearson Correlation Coefficient between DEGREE and the final grade in Physics 221, in spite of having a small value (0.21913), is significant at 0.0338.

The theoretical model designed and tested by Heller and Reif (1984), with the knowledge and procedures necessary for human problem solvers to generate good representations of scientific problems, allowed subjects to construct improved representations. These investigators hold that problem-solving deficiencies exist in students who understand basic physics concepts but do not have the more strategic knowledge specified in the formulated model. This kind of knowledge, possessed by experts, according to Heller and Reif (1984), is seldom taught explicitly in physics courses.

The DEGREE variable involves categorization which is linked to representation. If knowledge possessed by experts (including the ability to form good representations) is seldom taught explicitly, a failure in finding relationships between the final grades in Physics 221 and the attained scores on the DEGREE variable seems reasonable. The novice sample, however, includes 12 % who functioned like experts and 18 % who functioned like the subjects in the Silver (1981) study. It seems equally reasonable to assume that a given amount of expert behavior is taught (implicitly or explicitly) in Physics 221. The use of multiple-choice examinations, with their limitations in testing strategic knowledge, in Physics 221 is a more prosaic explanation of the absence of a grade-DEGREE relationship.

Return to the Research Question

Do novices and experts differ in the categorization of physics (mechanics) problems?

The findings of this research confirm the Chi et al. (1981) results regarding experts: Experts categorize according to deep structures.

The behavior of novices is more complex. Novices use both surface features and deep structures in the categorization process. Novices demonstrate a lesser degree of consistency in the categorization process. Approximately one third of the novices demonstrate expert behavior.

The inclusion of larger numbers of subjects than is customary in this kind of research has resulted in a greater degree of generalization and has revealed the more complex behavior of novices as they categorize mechanics problems. The use of dendograms and the DEGREE variable allow for an increase in reproducibility.

Suggestions for Further Research

1. This study found marked differences in DEGREE scores attained by the novices. It was inferred that such differences are indicative of differences in the schemata of the novices. A longitudinal study investigating when and how such differences originate may clarify the categorization process in ways that would allow for classroom testing and use of theoretical models such as that by Heller and Reif (1984), discussed in the problem-representation section of Chapter II.
2. The differences in DEGREE scores and the final grades in Physics 221 ($r = 0.21913$ at a significance level of 0.0338) can be investigated by the replication of this study with the accompanying use of tests in which subjects solve problems of the the types used in this study. These tests may be constructed in

order to serve the purposes of such an investigation and the evaluative program in a calculus-based physics course.

3. Some of the problems used in this study are typical of problems that are often used as examples in physics texts for the introduction of topics such as the conservation of mechanical energy. Other are similar to problems commonly assigned for homework. Problems 1.5 and 2.6, discussed in some detail earlier in this chapter, are examples of such types. Students may indeed mimic deep structures by merely associating some of the problems used in this study with particular sections of physics texts. A study including more difficult problems that are not ordinarily used in lecture or homework assignments may be designed to establish or unmask such mimicry.

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The Committee on the Use of Human Subjects in Research at Iowa State University, upon having reviewed this study, expressed that the rights and privileges of the subjects were properly observed, that the potential value of the study exceeded the risks, that informed consent was obtained appropriately, and that confidentiality of data was assured.

APPENDIX A: PROBLEM REFERENCES AND PROBLEMS - PROBLEM SET 1

PROBLEM REFERENCES AND PROBLEMS - PROBLEM SET 1

1. Problem Reference

Halliday, D. & Resnick, R. Fundamentals of Physics. New York: John Wiley & Sons 1974.

2. Problems

1.1 - Two blocks of mass M(1) and M(2) are attached by a spring. They rest on a frictionless surface. If the two are given velocities such that the first travels at velocity V(1) towards the center of mass, which remains at rest, what is the velocity of the second? (p. 149).

1.2 - A girl (mass M) stands on the edge of a merry-go-round (mass 10M, radius R, rotational inertia I) that is not moving. She throws a rock (mass m) in a horizontal direction, tangent to the outer edge of the merry-go-round. The speed of the rock relative to the ground is V. Neglecting friction, what is the angular speed of the merry-go-round? (p. 208a).

1.3 - A block of mass M(1) is put on a plane inclined at an angle θ to the horizontal and is attached by a cord parallel to the plane over a pulley at the top to a hanging block of mass M(2). Block M(2) falls a distance x in time T. The pulley has a mass M(3) and a radius R, and can be considered to be a uniform disk. What is the coefficient of friction between the block and the plane? (p. 208).

1.4 - A hunter has a rifle that can fire 60 gm bullets with a muzzle velocity of 900 m/sec. A 40 kg leopard springs at him with a speed of 10 m/sec. How many bullets must the hunter fire into the leopard in order to stop him in his tracks? (p. 151).

1.5 - A block of mass M starts up an incline of angle θ with respect to the horizontal, with an initial velocity V. How far will it slide up the plane if the coefficient of friction is μ? (p. 131).

1.6 - A block of mass M(1) slides along a frictionless table with a velocity V(1). Directly in front of it, and moving in the same direction, is a block of mass M(2) moving at a velocity V(2), where V(2) is less than V(1). A massless spring with a spring constant K is attached to the backside of M(2). When the blocks collide, what is the maximum compression of the spring? (p. 171).

1.7 - A horizontal spring of negligible mass and length L(1) is fastened to a wall. A block of mass M is forced against it, compressing the spring to length L(2). When the block is released, it moves a distance x across a horizontal surface before coming to rest. The force constant of the spring is K. What is the coefficient of sliding friction between the block and the table? (pp. 108, 109).

1.8 - A block of mass M(1) is put on top of a block of mass M(2). In order to cause the top block to slip on the bottom one, a horizontal force F(1) must be applied to the top block. Assume a frictionless table. Find the maximum horizontal force F(2) which can be applied to the lower block so that the blocks will move together (p. 94).

1.9 - Two disks are connected by a stiff spring, one disk directly above the other. Can one press the upper disk down enough so that when it is released it will spring back and raise the lower disk off the table? (p. 128).

1.10 - A bullet of mass M(1) is fired horizontally into a wooden block of mass M(2) at rest on a horizontal surface. The coefficient of kinetic friction between block and surface is μ. The block moves a distance L before coming to rest again. Find the speed of the bullet (pp. 169, 170).

1.11 - A man of mass M(1) lowers himself to the ground from a height x by holding onto a rope passed over a massless, frictionless pulley and attached to a block of mass M(2). The mass of the man is greater than the mass of the block. What is the tension in the rope? (p. 91).

1.12 - A block of mass M hangs from a cord C which is attached to the ceiling. Another cord D is attached to the bottom of the block. Explain why, if you jerk suddenly on D it will break, but if you pull steadily on D, C will break. (p. 86).

1.13 - A wheel is rotating with an angular speed ω on a shaft whose rotational inertia is negligible. A second wheel, initially at rest and with twice the rotational inertia of the first is suddenly coupled to the same shaft. How does the rotational kinetic energy of the system change? (p. 208a).

1.14 - A block of mass M(1) is put on a plane inclined at an angle θ to the horizontal and is attached by a cord parallel to the plane over a pulley at the top to a hanging block of mass M(2). The pulley has a mass M(3) and a radius R, and can be considered to be a uniform disk. The coefficient of kinetic friction between the block and plane is μ. Find the tension in the cord on each side of the pulley (p. 208).

1.15 - A small coin of mass M is placed on a flat horizontal turntable rotating at angular velocity ω. The coin is observed to slide off when at a distance from the center of the turntable greater than R. What is the coefficient of static friction between the coin and the turntable? (p. 149).

1.16 - A block of mass M is dropped from a height x onto a spring of force constant K. Neglecting friction, what is the maximum distance the spring will be compressed? (p. 129).

1.17 - Two particles, one having N times the mass of the other, are held together with a compressed spring between them. There is an amount of energy E stored in the spring. How much kinetic energy does each have after they are released? (p. 170).

1.18 - Two blocks of mass M(1) and M(2) are attached by a spring. They rest on a frictionless surface. Find the ratio of their accelerations A(1) and A(2) after they are pulled apart and then released (p. 88).

1.19 - A man of mass M(1) lowers himself to the ground from a height x by holding onto a rope passed over a massless, frictionless pulley and attached to another block of mass M(2). The mass of the man is greater than the mass of the block. With what speed does the man hit the ground? (p. 91).

1.20 - A man of mass M(1) lowers himself to the ground from a height x by holding onto a rope passed over a pulley and attached to another block of mass M(2). The pulley has a mass M(3) and a radius R and can be considered to be a uniform disk. The mass of the man is greater than the mass of the block. With what speed does the man hit the ground? (p. 91).

1.21 - A bullet of mass M(1) strikes a ballistic pendulum of mass M(2). The center of mass of the pendulum rises a vertical distance x. Assuming the bullet remains embedded in the pendulum, calculate its initial speed (p. 170).

1.22 - Two blocks of masses 1 kg and 3 kg connected by a spring rest on a frictionless surface. If the two are given velocities such that the first travels at 1.7 meters/sec toward the center of mass, which remains at rest, what is the velocity of the second? (p. 149).

1.23 - A 2 kg block is forced against a horizontal spring of negligible mass, compressing the spring by 15 cm. When the block is released, it moves 60 cm across a horizontal tabletop before coming to rest. The force constant of the spring is 200 nt/meter. What is the coefficient of sliding friction between the block and the table? (pp. 108, 108a).

1.24 - The spring of a spring gun has a force constant K. When the gun is inclined at an angle θ to the horizontal, a ball of mass M is projected to a height x. By how much must the spring have been compressed initially? (p. 132).

APPENDIX B: PROBLEM REFERENCES AND PROBLEMS - PROBLEM SET 2

PROBLEM REFERENCES AND PROBLEMS - PROBLEM SET 2

1. Problem References

Fried, R. Introductory Physics. Boston: Allyn and Bacon, 1966.

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Weidner, R. T. & Sells, R. L. Elementary Classical Physics. Boston, MA: Allyn and Bacon, 1973.

2. Problems

2.1 - An object of mass M is suspended from the end of a light cord. The cord is pulled to the side by a horizontal force until the angle between the cord and the vertical is 30 degrees. What is the magnitude of the horizontal force? (Weidner & Sells, 1973, p. 127).

2.2 - The force required to compress a horizontal spring an amount x is given by F = ax + b(x to the third power) where a and b are constants. If the spring is compressed an amount l, what speed will it give to a ball of mass M held against it and released? (Giancoli, 1984, p. 113).

2.3 - A uniform drum of radius b and mass M rolls without slipping down a plane inclined at an angle θ. The moment of inertia of the drum about its axis is I(0) = M (R to the second power)/2. Find the acceleration of the drum along the plane (Kleppner & Kalenkow, 1973, p. 265).

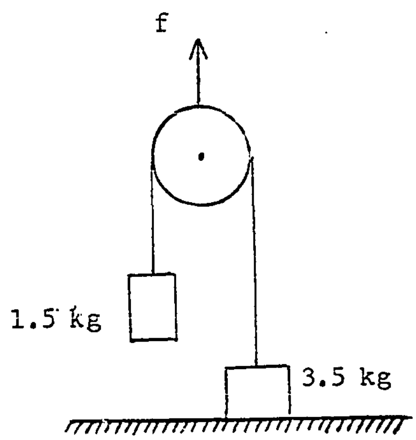
2.4 - A 1.0 kg ball is dropped from 1.5 m above a floor. It rebounds to 1.0 m. The ball is in contact with the floor for 1.0×10^{-3} s. What is the direction and magnitude of the average force on the ball over the entire motion? (Weidner & Sells, 1973, p. 105).

2.5 - A block of mass M starts up an incline of angle θ with respect to the horizontal, with an initial velocity V. How far will it slide up the plane if the coefficient of friction is μ? (Halliday & Resnick, 1974, p. 131).

2.6 - A child of mass M descends a slide of height h and reaches the bottom with a speed of v. Calculate the amount of heat generated in the process (Giancoli, 1984, p. 162).

2.7 - A horizontal spring of negligible mass and length L(1) is fastened to a wall. A block of mass M is forced against it, compressing the spring to length L(2). When the block is released, it moves a distance x across a horizontal surface before coming to rest. The force constant of the spring is K. What is the coefficient of sliding friction between the block and the table? (Halliday & Resnick, 1974, pp. 108, 108a).

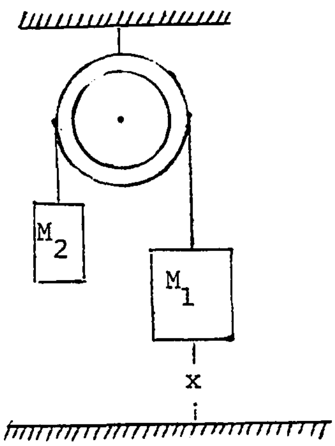
2.8 - Two bodies of mass 1.5 and 3.5 kg are attached to opposite ends of a massless string which passes over a pulley as shown. Taking g to be 10 m/s^2, find the maximum and minimum values of the upward external force f on the pulley such that the 3.5 kg mass will remain at rest on the table and the string remain taut (Weidner & Sells, 1973, p. 129).



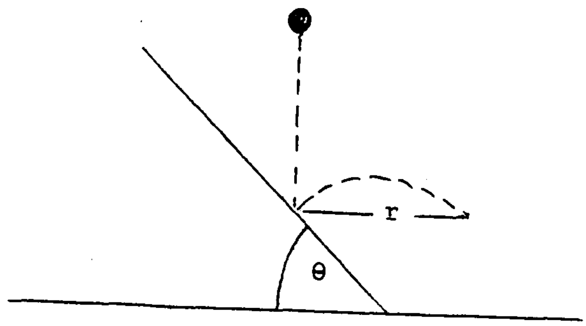
2.9 - Two ice skaters, each of mass M, are traveling in opposite directions with speed V but separated by a distance L perpendicular to their velocities. When they are just opposite each other, each grabs one end of a rope of length L. Each now pulls in on her end of the rope until the length of the rope is L/2. What is the speed of each skater? (Kittel, Knight, & Ruderman, 1973, p. 199).

2.10 - A helicopter has a main rotor that rotates in a horizontal plane about a vertical axis. It has a small auxiliary rotor at the tail that rotates in a vertical plane about a horizontal axis. What is the function of the auxiliary rotor? (Miller, 1982, p. 165).

2.11 - A man of mass M(1) lowers himself to the ground from a height x by holding onto a rope passed over a massless, frictionless pulley and attached to a block of mass M(2). The mass of the man is greater than the mass of the block. What is the tension in the rope? (Halliday & Resnick, 1974, p. 91).



2.12 - A ball is dropped onto a solid, firmly anchored, smooth inclined plane with which it makes a perfectly elastic collision. The angle θ of the plane with the horizontal is adjusted so that the range r of the rebound trajectory is a maximum. What is the value of θ? (Strong, 1972, p. 275).



2.13 - A wheel is rotating with an angular speed ω on a shaft whose rotational inertia is negligible. A second wheel, initially at rest and with twice the rotational inertia of the first is suddenly coupled to the same shaft. How does the rotational kinetic energy of the system change? (Halliday & Resnick, 1974, p. 208a).

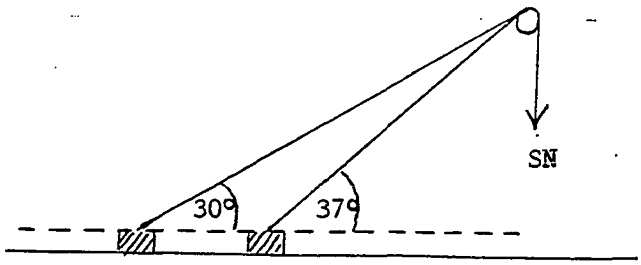
2.14 - A tractor is pulling a heavy log in a straight line. One might argue that the log pulls back on the tractor just as strongly as the tractor pulls on the log. Explain why the tractor indeed does move (Rutherford, Holton, & Watson, 1981, p. 98).

2.15 - A bullet of mass M is shot through a weather vane having a moment of inertia I relative to its axis and originally at rest. The bullet misses the axis of the weather vane by a distance d, and its speed is reduced from v to 1/4v in passing through. What is the angular speed of the weather vane after the bullet passes through? (Weidner & Sells, 1973, p. 205).

2.16 - A particle of mass m slides down a frictionless incline of angle α, mass M, and length L which is on a frictionless plane. If the particle starts initially from rest at the top of the incline, prove that the time for the particle to reach the bottom is given by

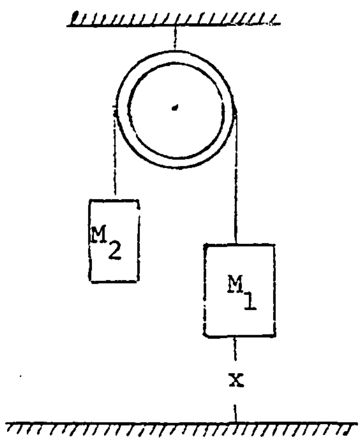
$$\sqrt{\frac{2\,L\,(M + m\,\sin\;\;)}{(M + m)\,g\,\sin}}$$ (Spiegel, 1967, p. 213).

2.17 - A string attached to a block of mass 1.0 kg, initially at rest on a horizontal frictionless surface passes over a frictionless pulley as shown. A force of constant magnitude, 5N, is applied to the string, the block thereby being accelerated to the right. Find the work done by this force on the block, when the block moves from the point where the string makes an angle of 30 degrees with the horizontal to the point where the string makes an angle of 37 degrees to the horizontal. The pulley is 1.0 m above the top of the block (Weidner & Sells, 1973, p. 149).



2.18 - Two blocks of mass M(1) and M(2) are attached by a spring. They rest on a frictionless surface. Find the ratio of their accelerations A(1) and A(2) after they are pulled apart and then released (Halliday & Resnick, 1974, p. 88).

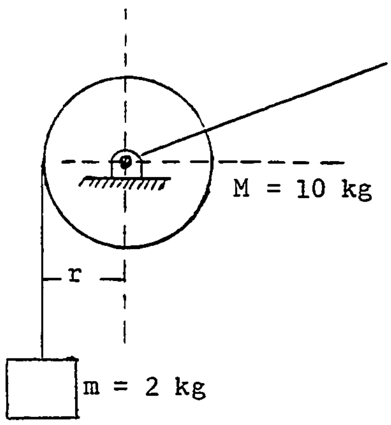
2.19 - A man of mass M(1) lowers himself to the ground from a height x by holding onto a rope passed over a massless, frictionless pulley and attached to another block of mass M(2). The mass of the man is greater than the mass of the block. With what speed does the man hit the ground? (Halliday & Resnick, 1974, p. 91).



2.20 - A 70 kg man standing at rest on frictionless ice sees a 10 kg object sliding towards him from the north at 5.0 m/s. He catches it and throws it southward so that it again travels south at 5.0 m/s. What is the man's final velocity? (Weidner & Sells, 1973, p. 74).

2.21 - A ladder of mass M and length L rests against a slippery vertical wall at an angle θ with the vertical. The ladder, of uniform construction, is prevented from slipping by friction with the ground. Calculate the magnitude of the force exerted by the ladder on the wall (Kittel, Knight, & Ruderman, 1973, p. 197).

2.22 - A light string carrying a 2 kg mass is wrapped around a 10 kg solid cylindrical disk of 20 cm radius, supported in frictionless bearings as shown. The system is released from rest. Calculate the angular speed of the disk at the end of 10 s (Fried, 1966, pp. 149-151).



2.23 - A rocket-fuel system of total mass M is at rest on a horizontal frictionless surface. What is the final velocity of the remaining rocket with respect to the surface if one shot of mass 3/5 M is fired to the left at speed v relative to the rocket? (Weidner & Sells, 1973, p. 75).

2.24 - What happens to the length of day when a sprinter starts running in an easterly direction? The sprinter runs along the surface of the rotating earth; the change in the length of day is too small to observe (Miller, 1982, p. 167).

APPENDIX C: PROBLEM REFERENCES AND PROBLEMS - PROBLEM SET 3

PROBLEM REFERENCES AND PROBLEMS - PROBLEM SET 3

1. Problem References

Giancoli, D. C. General Physics. Englewood Cliffs, NJ: Prentice-Hall, 1984.

Halliday, D. & Resnick, R. Fundamentals of Physics. New York: John Wiley & Sons, 1974.

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2. Problems

3.1 - If enough thermal energy to melt the polar icecaps were absorbed by the earth tomorrow, what would be the effect on the rotation of the earth? (Kyker, 1982, p. 88).

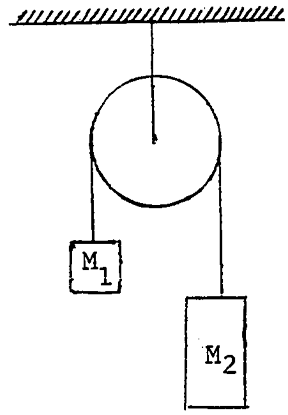
3.2 - The force required to compress a horizontal spring an amount x is given by F = ax + b(x to the third power) where a and b are constants. If the spring is compressed an amount l, what speed will it give to a ball of mass M held against it and released? (Giancoli, 1984, p. 113).

3.3 - A block of mass M(1) is put on a plane inclined at an angle θ to the horizontal and is attached by a cord parallel to the plane over a pulley at the top to a hanging block of mass M(2). Block M(2) acquires momentum as it falls a distance x in time T, causing the momentum of M(1) to change. The pulley has a mass M(3) and a radius R, and can be considered to be a uniform disk. What is the coefficient of friction between the block and the plane? (Halliday & Resnick, 1974, p. 208).

3.4 - Two blocks of mass M(1) and M(2) are attached by a spring. They rest on a frictionless surface. If the two are given velocities such that the first travels at velocity v(1) towards the center of mass, which remains at rest, what is the velocity of the second? (Halliday &

Resnick, 1974, p. 149).

3.5 - An apparatus consists of two masses which are suspended over a pulley by means of a cord with one mass on each end of the cord. M(2) is greater than M(1). Assuming the cord and pulley to be massless and frictionless, calculate the tension in the cord (Giancoli, 1984, p. 75).

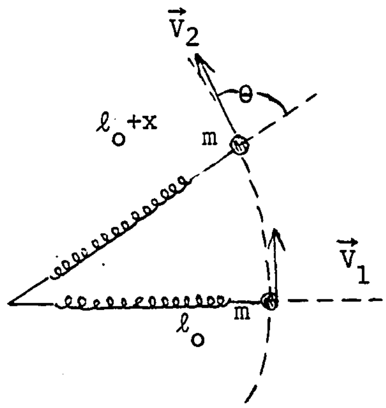


3.6 - A child of mass M descends a slide of height h and reaches the bottom with a speed of v. Calculate the amount of heat generated in the process (Giancoli, 1984, p. 132).

3.7 - The ballistic pendulum is a device used to measure the speed of objects such as bullets. The bullet, mass m, is fired into a large block of mass M that is suspended like a pendulum. As a result of the impact of the bullet, the kinetic energy of the bullet appears as the kinetic energy of the block and the embedded bullet. This kinetic energy is changed into potential energy of block and embedded bullet as the pendulum reaches maximum height, h. Calculate the speed of the block (mass M) and the embedded bullet (mass m) just after the collision if the bullet has an original speed v (Giancoli, 1984, p. 154).

3.8 - A child builds a machine that consists, among other things, of a number of pulleys and cords. After the child reluctantly has left the machine to go to bed, the mouse which had observed the building process finds that curiosity outweighs fear and climbs onto the machine. The mouse, of mass M, sits at the edge of a motionless pulley that may rotate freely, without friction, in a horizontal plane about a vertical axis. The pulley has a radius R and a moment of inertia I. The mouse now runs around the edge of the pulley in a clockwise direction and reaches an angular speed ω(mouse) with respect to the ground. What is the angular velocity (magnitude and direction) of the pulley?

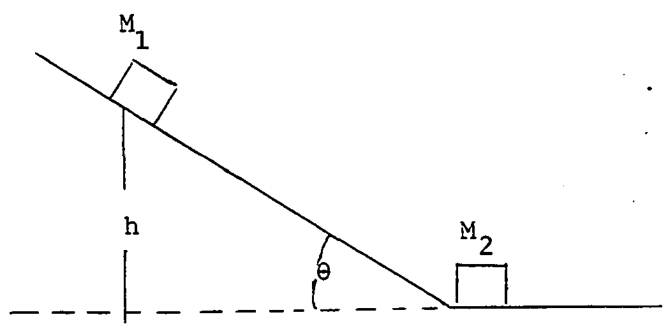
3.9 - A particle of mass m is attached to one end of a spring of relaxed length 1(0) and force constant K. The spring's other end is fixed in position. The particle slides over a horizontal frictionless surface. Initially the spring is relaxed, and the particle's velocity $\vec{v}(1)$ is at right angles to the long axis of the spring. At some later time the particle reaches point 2; here the spring's length is 1(0) + x and the particle's velocity is $\vec{v}(2)$, the direction between $\vec{v}(2)$ and the long axis of the spring being θ. Calculate the velocity $\vec{v}(2)$, magnitude v(2) and direction θ, in terms of v(1), 1((0), m, and the spring's extension x and stiffness K (Weidner & Sells, 1973, p. 193).



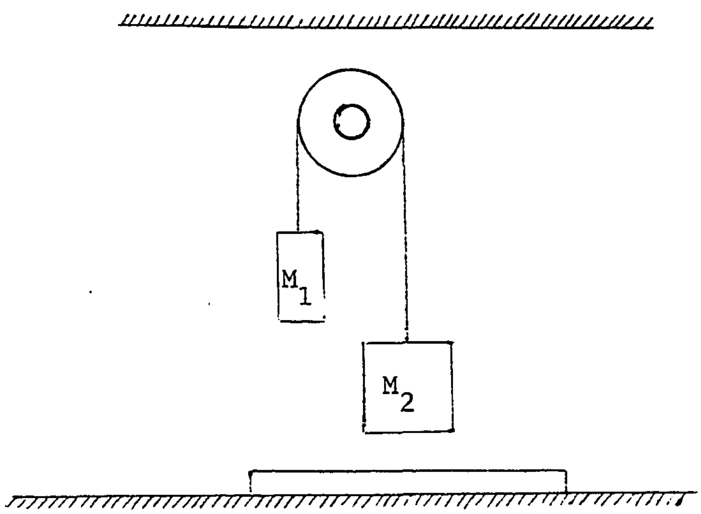
3.10 - A spring is kept compressed by tying its ends together tightly. The spring is placed in acid and dissolves. What happened to the potential energy of the spring? (Halliday & Resnick, 1981, p. 127).

3.11 - A body rests on an adjustable inclined plane. When the angle of the plane is increased from zero to some critical value θ, the body is on the point of sliding downward. Find an expression for the coefficient of static friction as a function of the angle (Morgan, 1964, pp. 91, 92).

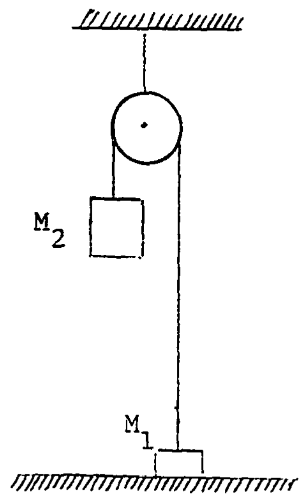
3.12 - A block of mass M(1) slides down an incline of height h that makes an angle θ with the horizontal. At the bottom it strikes a block of mass M(2) which is at rest on a horizontal surface. Assuming an elastic collision and negligible friction, determine the speeds after collision (Giancoli, 1984, p. 162).



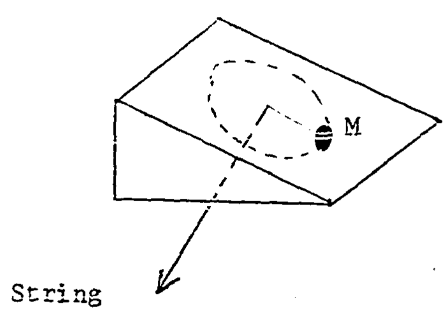
3.13 - An apparatus consists of two masses which are suspended over a pulley by means of a cord with one mass on each end of the cord. Assume that the pulley and cord are massless and frictionless. As M(2), which is greater than M(1), strikes a box on the floor of an elevator at rest, a certain impulse is delivered to the box by M(2). Now suppose that the elevator is moving upward at a constant speed v. How does this motion affect the impulse delivered to the box by M(2)? Explain.



3.14 - A block of mass M(2) and a block of mass M(1) are attached to opposite ends of a massless cord of length l. M(2) is greater than M(1). The cord is hung over a small frictionless and massless pulley a distance h from the floor, with M(1) initially at the floor level. Then the blocks are released from rest. What is the speed of either block when M(2) strikes the floor? (Weidner & Sells, 1973, p. 177).



3.15 - An object of mass M attached to the end of a string revolves in a circle on a frictionless inclined plane. The other end of the string passes through a hole in the incline. The string is at a right angle to the plane. Initially the ball rotates at an average speed v(1) in a circle of radius r(1). The string is then pulled slowly through the hole, still at a right angle, so that the radius is reduced to r(2). What is now the average speed v(2)? (Giancoli, 1984, p. 182).



3.16 - A ball of weight W(1) and a ball of weight W(2) are connected by a stretched spring of negligible mass. When the two balls are released simultaneously, the initial acceleration of the ball of weight W(1) is a(1) westward. What is the acceleration of the ball of weight W(2)? (Research and Education Assoc., 1976, p. 110).

APPENDIX D: PROBLEM REFERENCES AND PROBLEMS - PROBLEM SET 4

PROBLEM REFERENCES AND PROBLEMS - PROBLEM SET 4

1. Problem References

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2. Problems

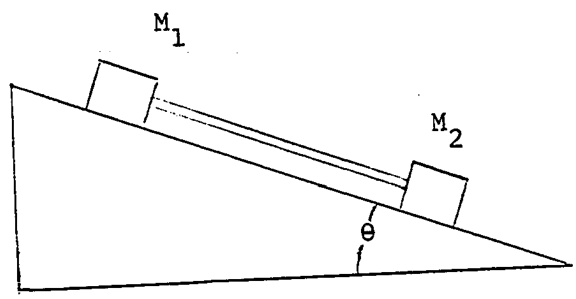
4.1 - A block slides down an inclined plane with an angle of incline θ. The coefficient of kinetic friction between the plane and the block is μ. Calculate the acceleration of the block (Halliday & Resnick, 1981, p. 88).

4.2 - The force required to compress a horizontal spring an amount x is given by F = ax + b(x to the third power) where a and b are constants. If the spring is compressed an amount l, what speed will it give to a ball of mass M held against it and released? (Giancoli, 1984, p. 113).

4.3 - A spring with a spring constant K is attached to the ceiling. A block of mass M is attached to its lower end. The spring is at a right angle to the ceiling. Calculate the tension in the spring.

4.4 - Two masses - M(1) and M(2) with M(2) greater than M(1) - attached by a massless rod, travel down along an inclined plane with an angle of incline θ. The rod is parallel to the plane. The coefficient of kinetic friction between M(1) and the incline is μ(1) and the coefficient of kinetic friction between M(2) and the incline is μ(2). Compute the tension in the rod linking M(1) and M(2) (Halliday &

Resnick, 1978, p. 113).



4.5 - A block of mass M starts up an incline of angle 0 with respect to the horizontal, with an initial velocity V. How far will it slide up the plane if the coefficient of friction is μ? (Halliday & Resnick, 1974, p. 127).

4.6 - A child of mass M descends a slide of height h and reaches the bottom with a speed of v. Calculate the amount of heat generated in the process (Giancoli, 1984, p. 162).

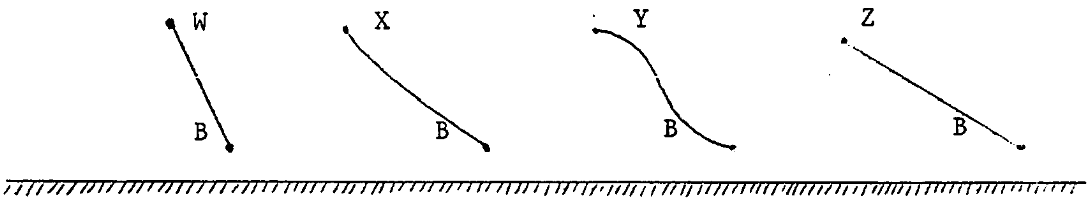
4.7 - A horizontal spring of negligible mass and length L(1) is fastened to a wall. A block of mass M is forced against it, compressing the spring to length L(2). When the block is released, it moves a distance x across a horizontal surface before coming to rest. The force constant of the spring is K. What is the coefficient of sliding friction between the block and the table? (Halliday & Resnick, 1974, pp. 108, 108a).

4.8 - A body of mass M is held in position on a frictionless inclined plane by a cable that is parallel to the plane. The angle of the inclined plane is θ. Calculate the tension in the cable (Tipler, 1982, p. 113).

4.9 - An object of mass M, attached to one end of a horizontal, compressed spring of negligible mass, rests on a frictionless horizontal surface. The other end of the spring is fixed in position. The spring is now released. Show that the acceleration of the object is proportional to its displacement (while in opposite direction from its displacement) with the constant of proportionality being K/M where K is the spring constant.

4.10 - In an experiment to determine the coefficient of friction a block of mass M is pulled at constant speed along a horizontal table top by means of a spring of spring constant K. If the length of the spring increases from L(1) to L(2) when the block is pulled, what is the coefficient of friction? (Lehrman & Swartz, 1969, p. 142).

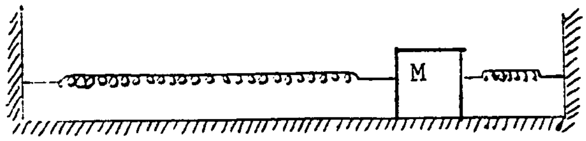
4.11 - A girl wants to slide down a playground slide so that she will have the greatest possible speed when she reaches the bottom (point B). Discuss which of the pictured frictionless inclines she should choose. W, X, Y, Z are all a distance y above the ground and B is a distance d above the ground (Rutherford, Holton, & Watson, 1970, p. 13).



4.12 - Two disks are connected by a stiff spring, one disk directly above the other. Can one press the upper disk down enough so that when it is released it will spring back and raise the lower disk off the table? (Halliday & Resnick, 1974, p. 128).

4.13 - A block of mass M(1) is put on a plane inclined at an angle θ to the horizontal and is attached by a cord parallel to the plane over a pulley at the top to a hanging block of mass M(2). Block M(2) falls a distance x in time T. The pulley has a mass M(3) and a radius R, and can be considered to be a uniform disk. What is the coefficient of friction between the block and the plane? (Halliday & Resnick, 1974, p. 208).

4.14 - A body of mass M is attached to two springs along a line as shown. The force constants of the springs are K(1) and K(2). Each spring is stretched from its equilibrium position. Find the ratio of the amounts of stretching of the springs (Tipler, 1982, p. 152).



4.15 - A car of mass M is coasting down a hill with an angle of inclination θ. At a time when the car's speed is V the driver applies the brakes. What force F, parallel to the road, must be applied by the brakes if the car is to stop after traveling a distance d? (Schaum, 1961, p. 53).

4.16 - A block of mass M(1) slides along a frictionless table with a velocity V(1). Directly in front of it, and moving in the same direction, is a block of mass M(2) moving at a velocity V(2), where V(2) is less than V(1). A massless spring with a spring constant K is attached to the backside of M(2). When the blocks collide, what is the maximum compression of the spring? (Halliday & Resnick, 1974, p. 171).